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UNIVERSITÀ DEGLI STUDI
DI TRENTO



Remote synchronization: detailed account of a peculiar pattern-formation mechanism

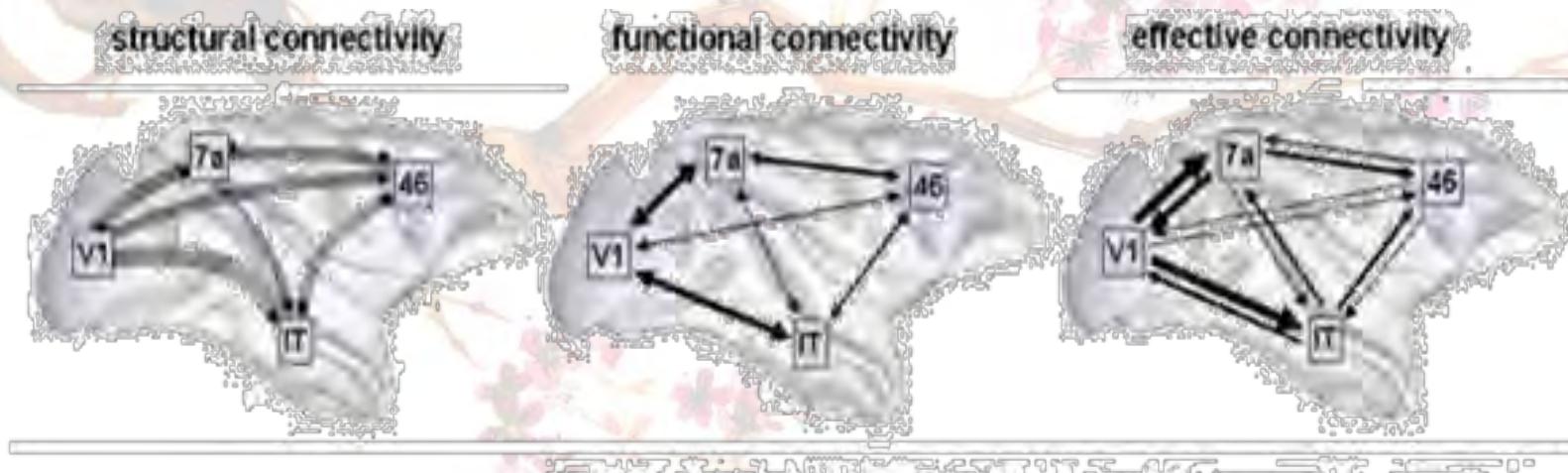
Ludovico Minati

Tokyo Institute of Technology, Tokyo, Japan
Institute of Nuclear Physics - Polish Academy of Sciences (IFJ-PAN), Kraków, Poland
University of Trento, Trento, Italy

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Context

A “rewarding” experiment about relationship(s) between structural connectivity and synchronization in an electronic network



What is remote synchronization?

Synchronised

A



B



C

Non-synchronised

Non-synchronised

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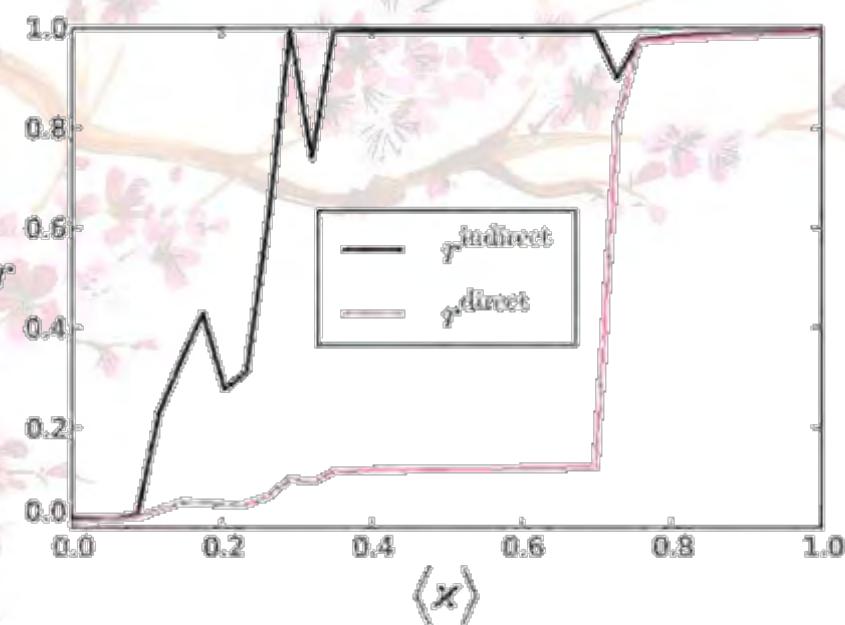
Remote synchronization from mismatches

PHYSICAL REVIEW E 85, 026208 (2012)

Remote synchronization in star networks

A. Bergner,^{1,3} M. Frasca,² G. Sciuto,² A. Buscarino,² E. J. Ngamga,³ L. Fortuna,² and J. Kurths^{3,4,5}

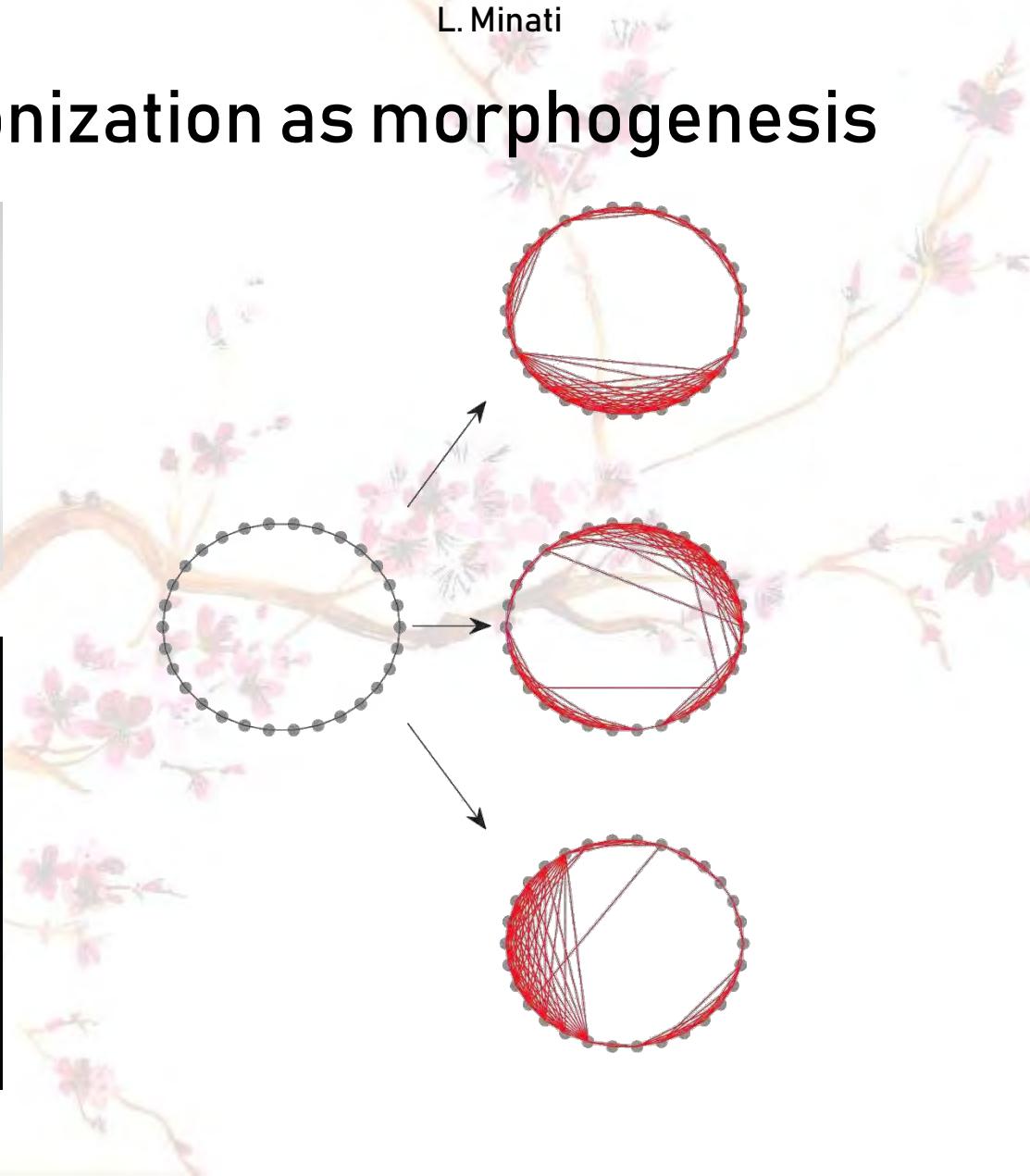
$$\dot{u}_i = (\alpha + i\omega_i - |u_i|^2)u_i + \frac{\sigma}{d_i} \sum_{j=1}^N a_{ij}(u_j - u_i)$$



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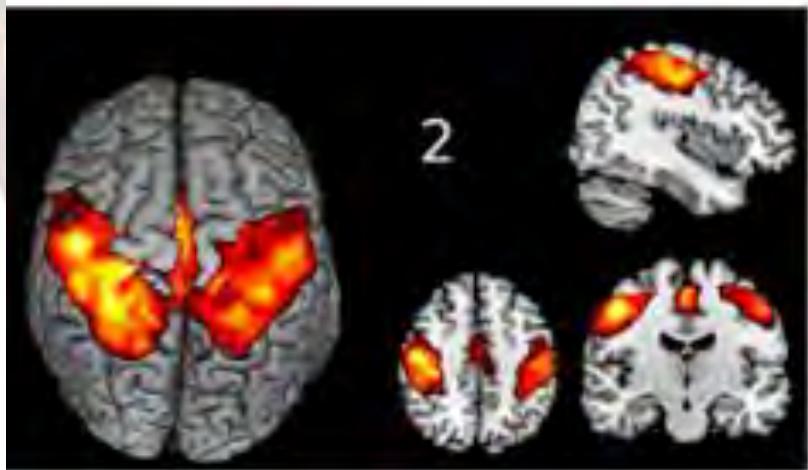
Remote synchronization as morphogenesis



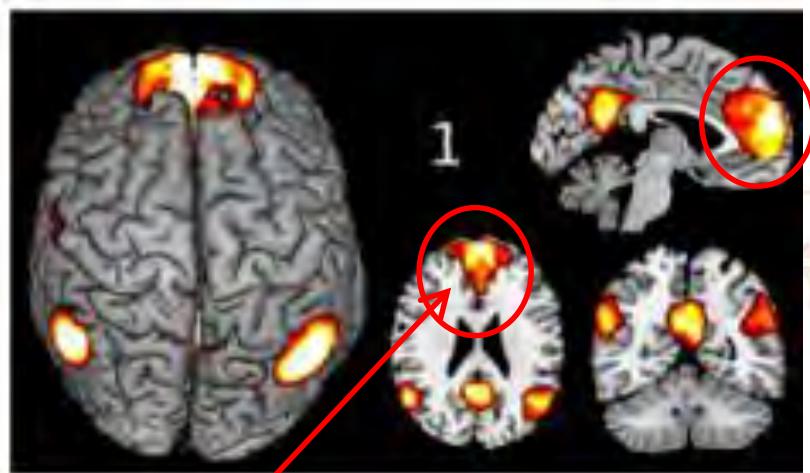
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Remote synchronization in brain networks?

Sensory-motor network: directly wired



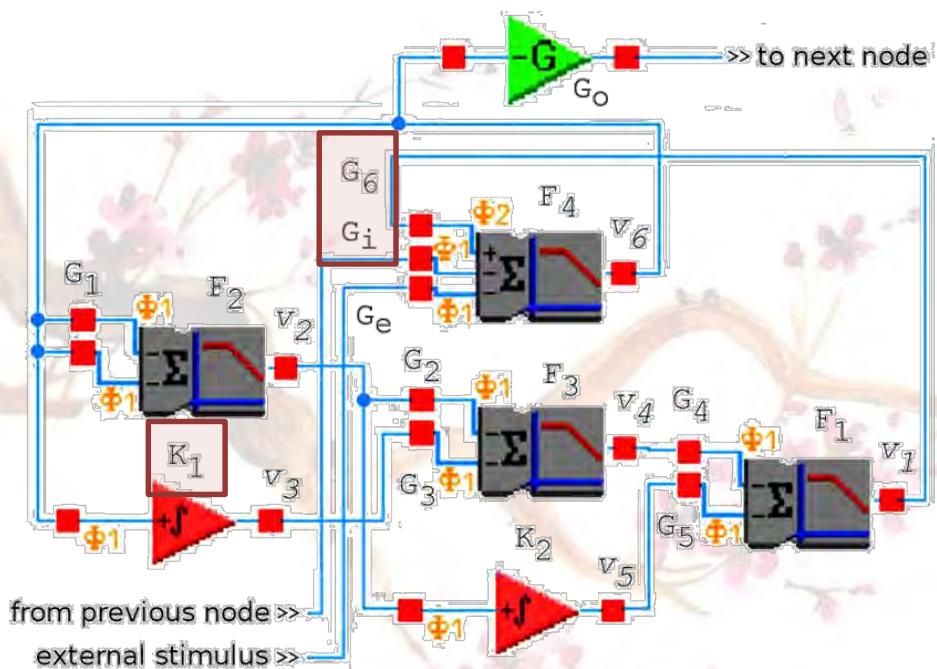
Default-mode network: emergent



No direct anatomical link to posterior areas.
Remotely synchronized?

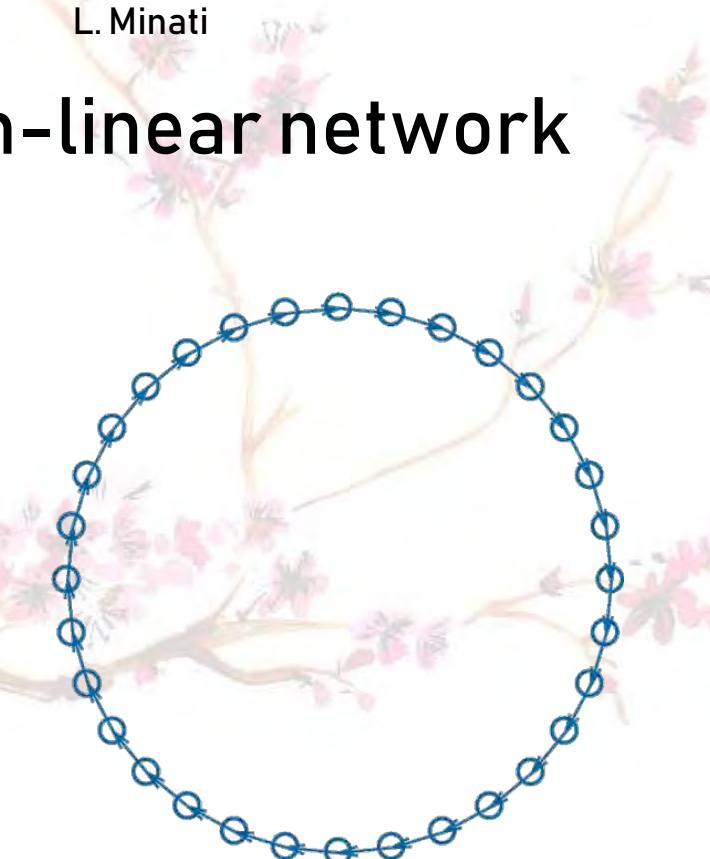
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A simple, reconfigurable non-linear network



a)

b)



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Oscillator equations

$$\begin{cases} \frac{dv_1}{dt} = \Gamma(2\pi F_1(G_4v_4 + G_5v_5 - v_1), v_1) \\ \frac{dv_2}{dt} = \Gamma(2\pi F_2(G_1v_6 - v_2), v_2) \\ \frac{dv_3}{dt} = \Gamma(K_1v_6, v_3) \\ \frac{dv_4}{dt} = \Gamma(2\pi F_3(G_2v_2 + G_3v_3 - v_4), v_4) \\ \frac{dv_5}{dt} = \Gamma(K_2v_2, v_5) \\ \frac{dv_6}{dt} = \Gamma(2\pi F_4(G_6v_1 + G_1v_1 + G_0v_0 - v_6), v_6) \end{cases}$$

Parametric mismatch
~0.5% in physical system

$$\Gamma(x, y) = R(x)H(V_s - y) - R(-x)H(V_s + y)$$

Applications in versatile pattern generation

Local Pattern Generators (LPGs)

Control parameters: P_3 , P_4 and P_5 .

Leg L1



Leg R2



Leg L3



Central Pattern Generator (CPG)

Control parameters: P_1 and P_2 .

L1/C

R3/C

R2/C

L2/C

L3/C

R1/C

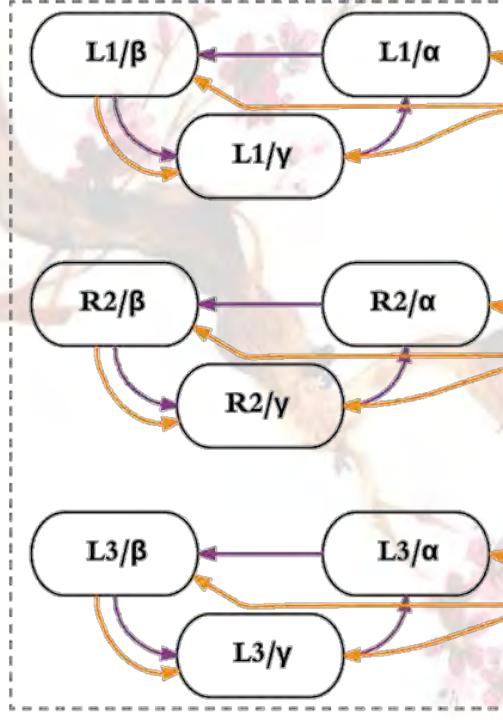
Local Pattern Generators (LPGs)

Control parameters: P_3 , P_4 and P_5 .

Leg R3

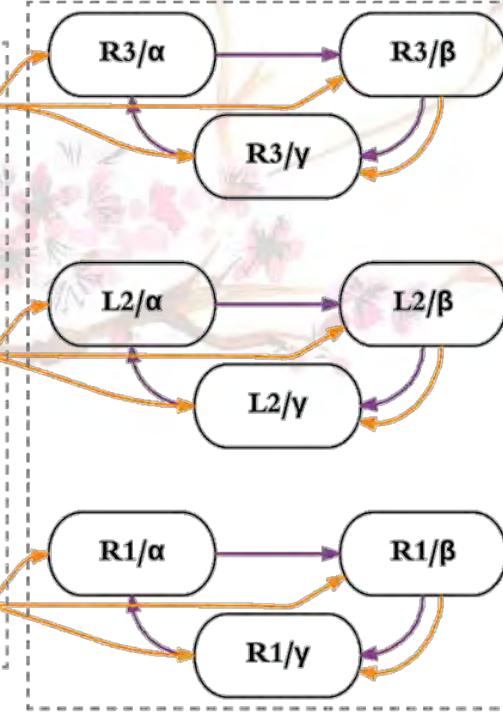
Leg L2

Leg R1



→ Extrinsic activity } P_3
 → Intrinsic activity } P_3

→ Extrinsic activity } P_1
 → Tetrapod gait (pos.) } P_1
 → Tripod gait (neg.) } P_1



→ Extrinsic activity } P_3
 → Intrinsic activity } P_3



Tokyo Tech



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Phase vs. amplitude synchronization

Phase coherence $r_{ij} = |\langle e^{i[\varphi_i(t) - \varphi_j(t)]} \rangle|$

Instantaneous amplitude (envelope)

$$v_i(t) + i\hat{v}_i(t) = A_i(t)e^{i\varphi_i(t)}$$

where \hat{v}_i is the Hilbert transform of $v_i(t)$

$$\hat{v}_i(t) = \frac{1}{\pi} \text{p.v.} \left[\int_{-\infty}^{\infty} \frac{v_i(\tau)}{t - \tau} d\tau \right]$$

and where p.v. denotes the Cauchy principal value of the integral¹⁸.

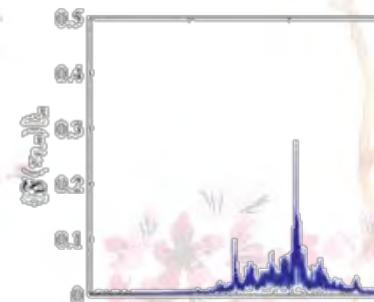
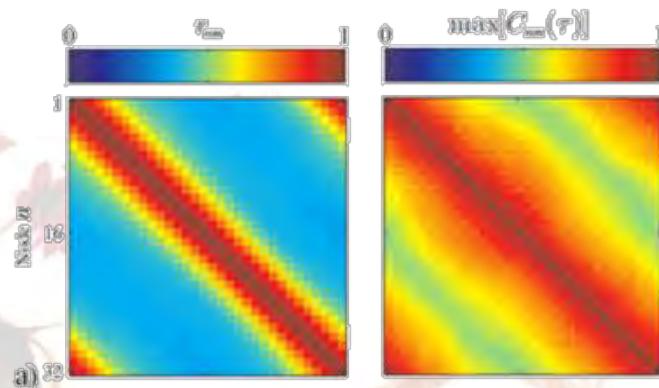
Maximum cross-correlation or mutual information

$$C_{XY}(\tau) = \frac{k_{XY}(\tau)}{\sqrt{\sigma_X^2 \sigma_Y^2}} \quad N_{XY}(d) = \frac{I_{XY}(d)}{\sqrt{H_X H_Y}}$$

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Numerical simulations reveal three regimes

$$a: G_6 = 0.196, \\ G_7 = -1.365$$

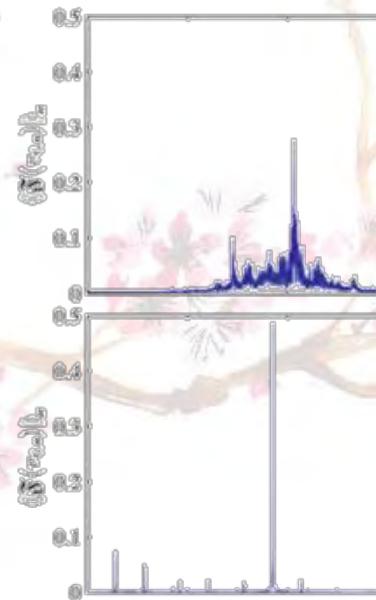
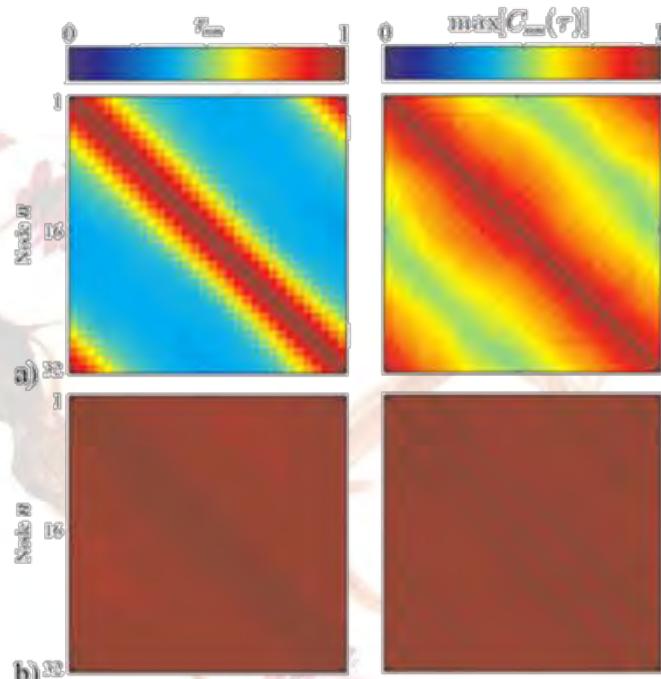


Broadband
chaos

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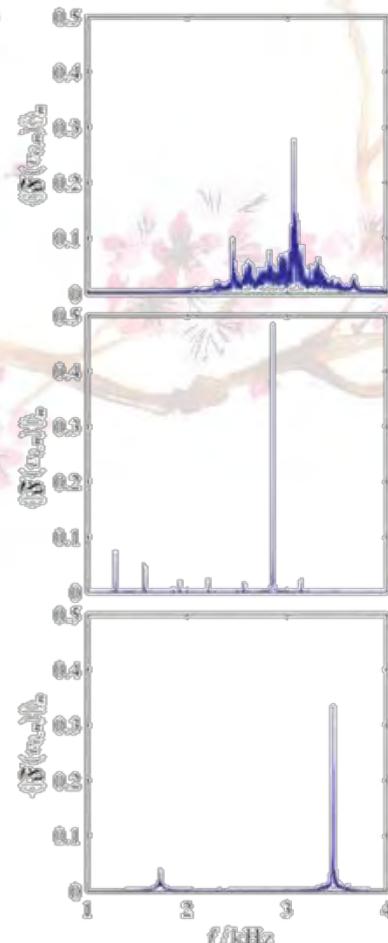
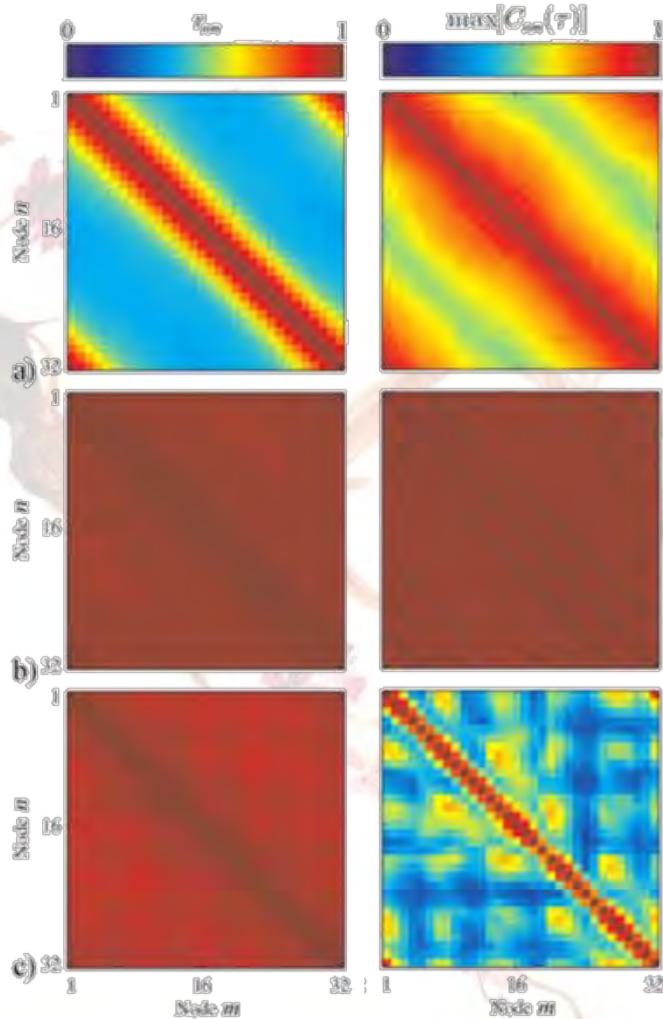
Numerical simulations reveal three regimes

$$\text{b: } G_6=0.096, \quad a: G_6=0.196, \\ G_7=-1.53 \quad G_7=-1.365$$

Broadband
chaosQuasi-
periodicity

Numerical simulations reveal three regimes

c: $G_6=0.188$, b: $G_6=0.096$, a: $G_6=0.196$,
 $G_7=-1.14$ $G_7=-1.53$ $G_7=-1.365$

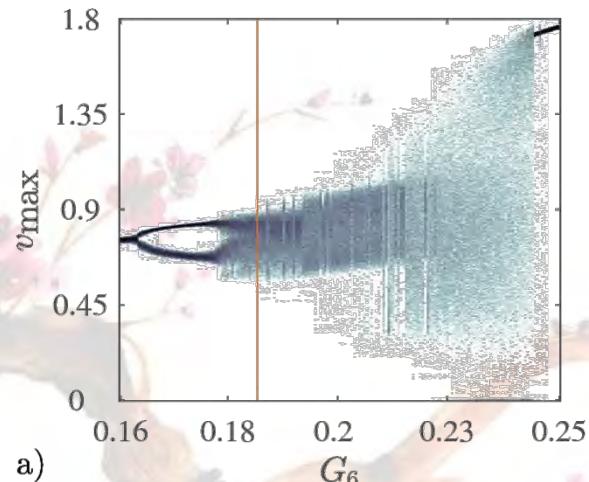


Broadband
chaos

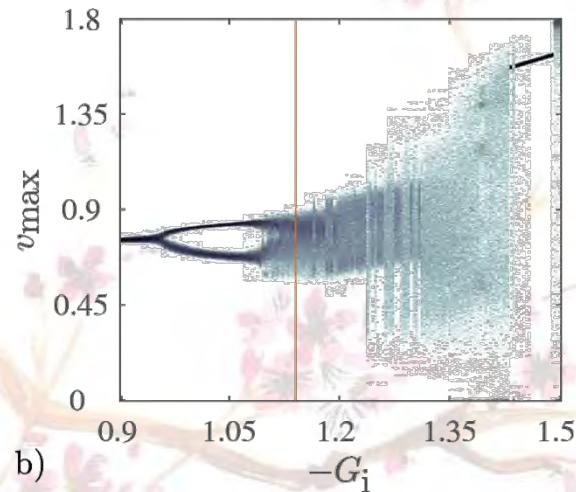
Quasi-
periodicity

Narrowband
chaos

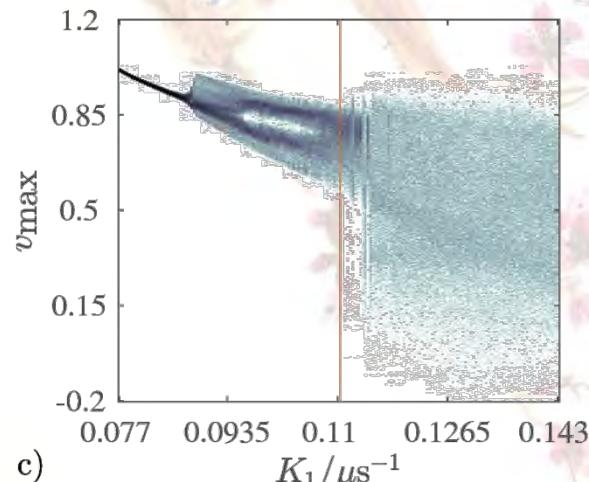
Numerical simulations reveal three regimes



a)



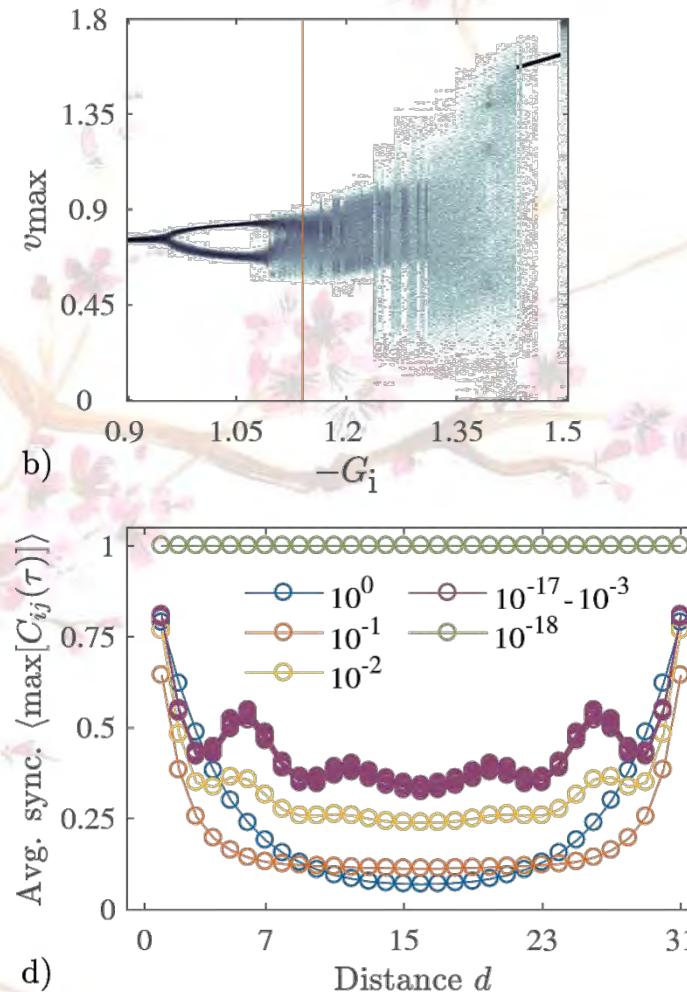
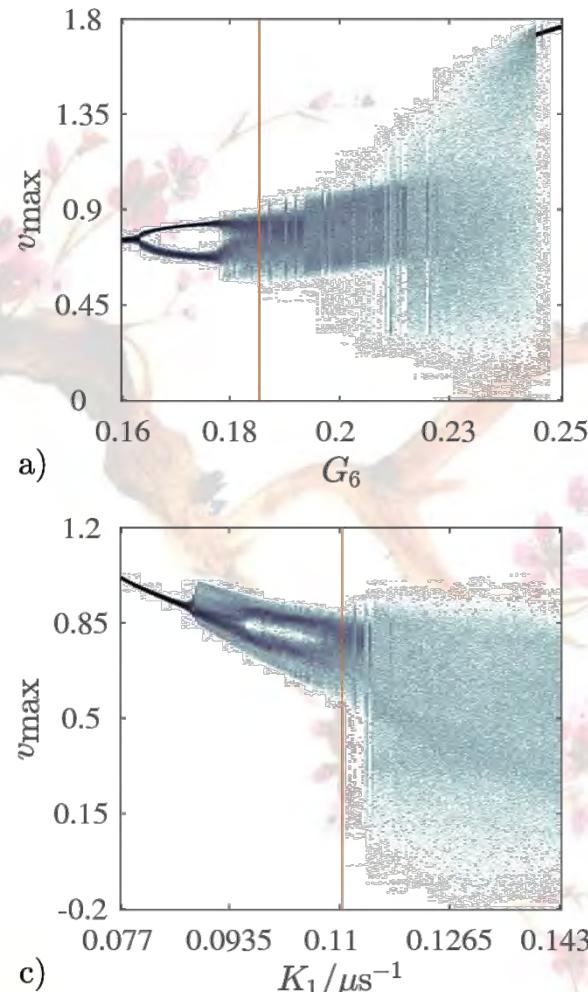
b)



c)

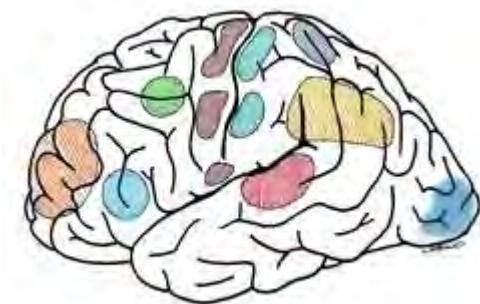
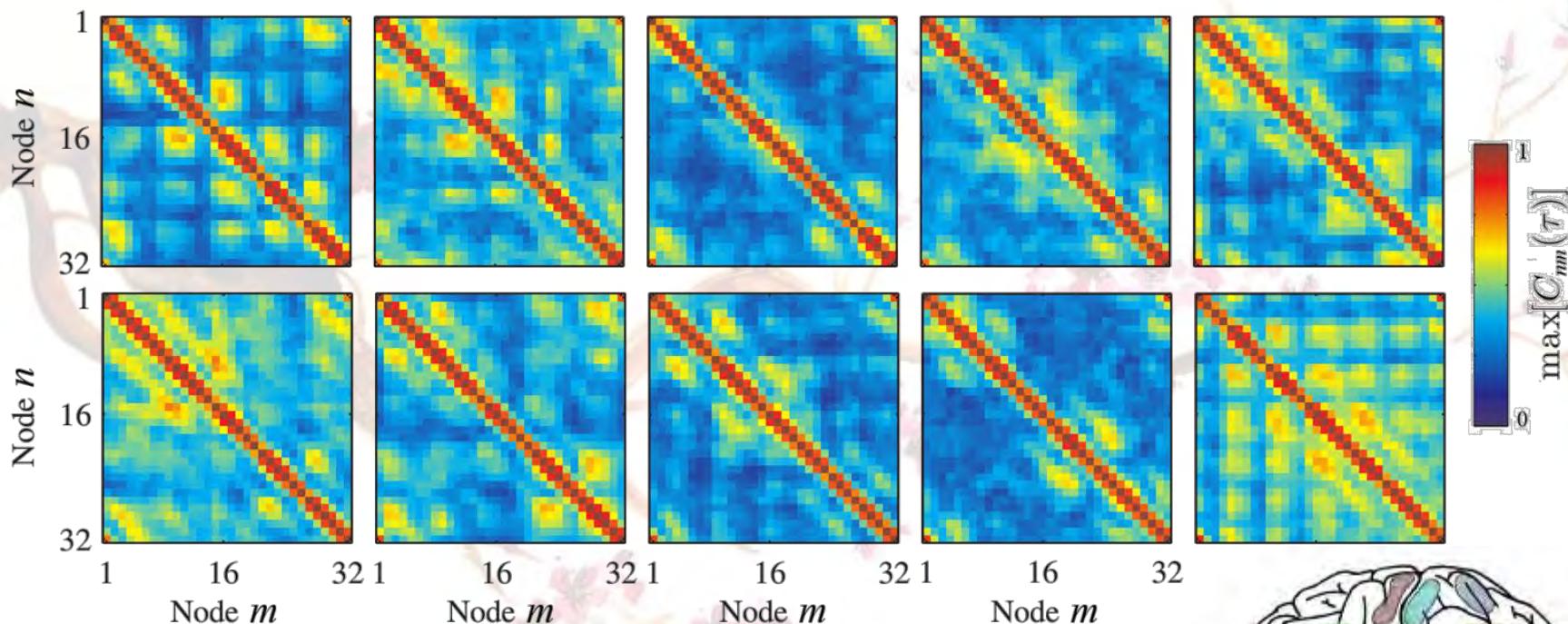
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Numerical simulations reveal three regimes



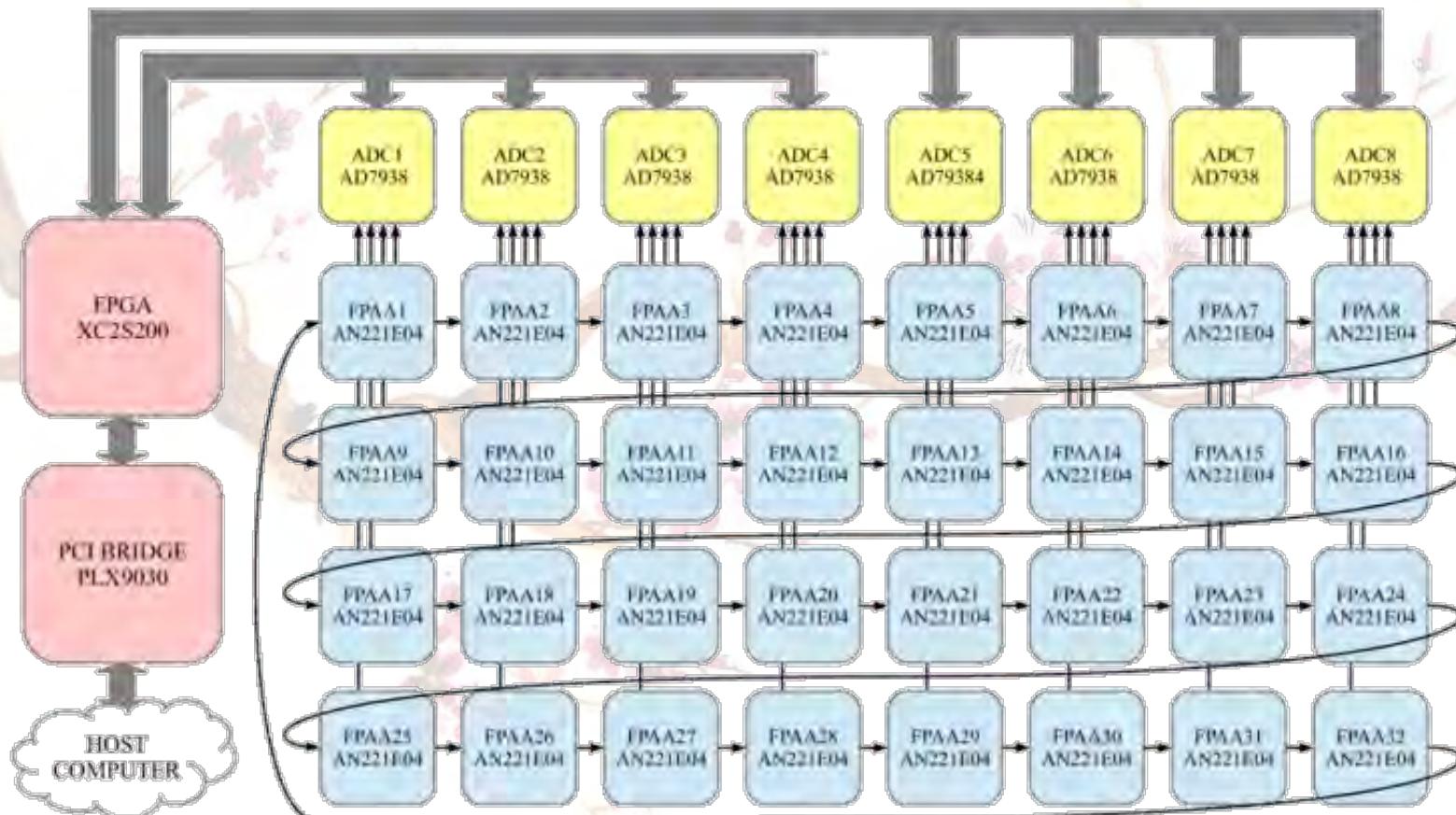
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Effect of parametric mismatches



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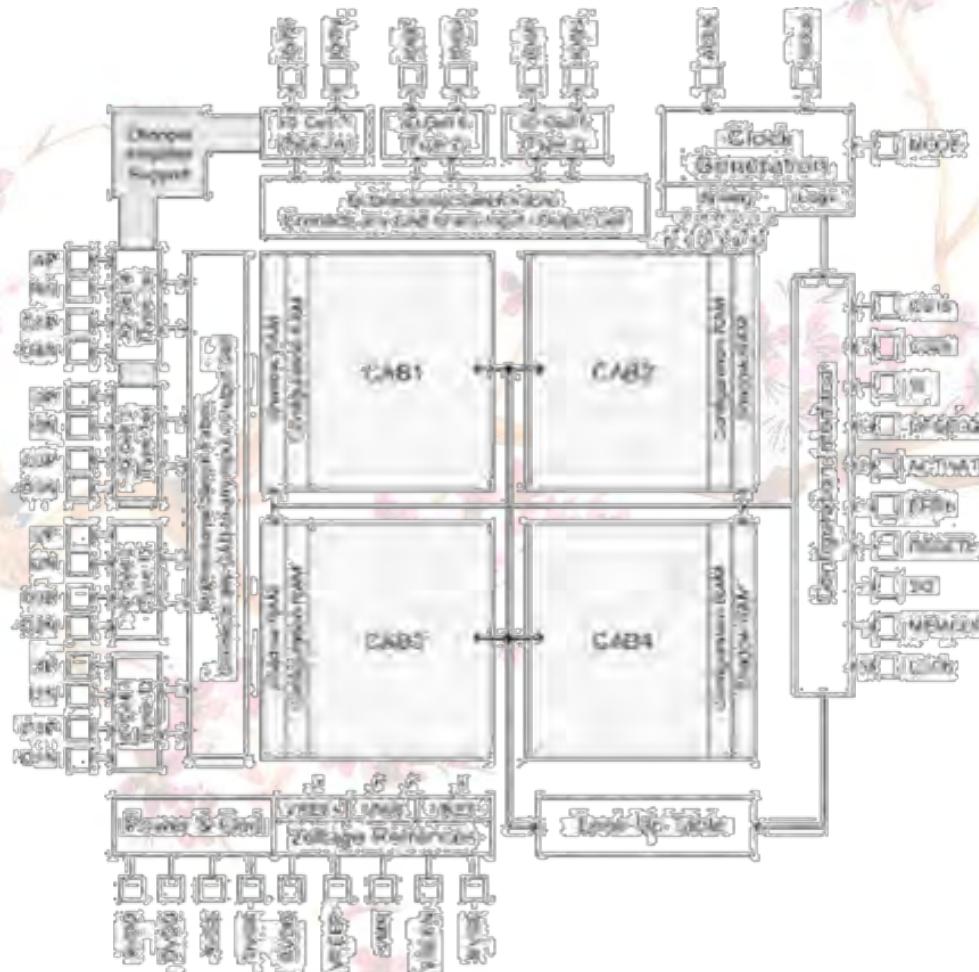
Experimental implementation



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Experimental implementation

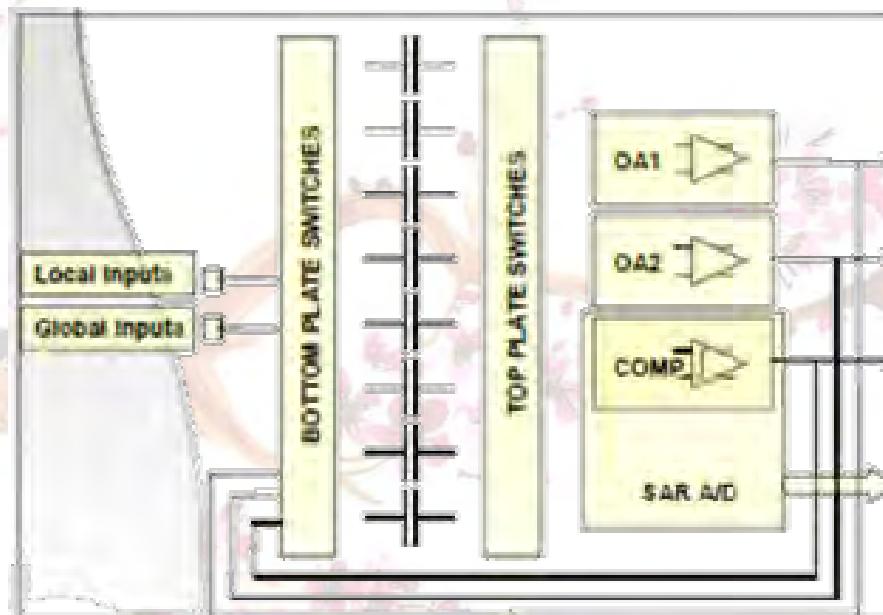
Top-level
FPGA
architecture



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Experimental implementation

The Configurable Analog Module (CAM)



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Experimental implementation

- **GainHalf** 

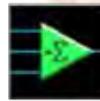
- Half-cycle

- **GainHold** 

- Inverting only

- **GainInv** 

- Continuous Time

- **SumInv** 

- Up to three inputs

- **SumDiff (SumHalf)** 

- Up to four inputs

- Add or subtract since input branches can be
inverting or non-inverting

- **RectifierFilter**

- Full Wave/Half Wave

- Inverting/non-inverting



- **RectifierHalf**

- Full Wave/Half Wave

- Inverting/non-inverting



- **RectifierHold**

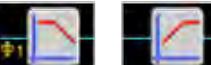
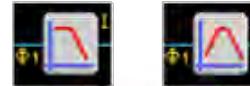


- Half Wave Inverting only



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Experimental implementation

- **FilterBilinear – One pole** 
 - Low Pass/High Pass/All Pass
- **FilterBiquad – Two poles** 
 - Low Pass/High Pass/Band Pass/Band Stop
 - Automatically chooses from multiple circuit topologies  
- **Differentiator** 
 - Output voltage slews – see documentation
- **Integrator** 
 - Optional reset 

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Experimental implementation

- **Comparator**

- Single/Dual Input



- Variable Reference



- **Hold – Sample and hold**



- **OscillatorSine**



- Subject to internal reference voltage error

- **Voltage (+/- 3 VDC)**



- Subject to internal reference voltage error

- **Multiplier**



- Uses SAR (Input Y is quantized)
 - Subject to internal reference voltage error
 - Optional sample and hold on input X to equalize sampling time of two inputs (uses chip resources)

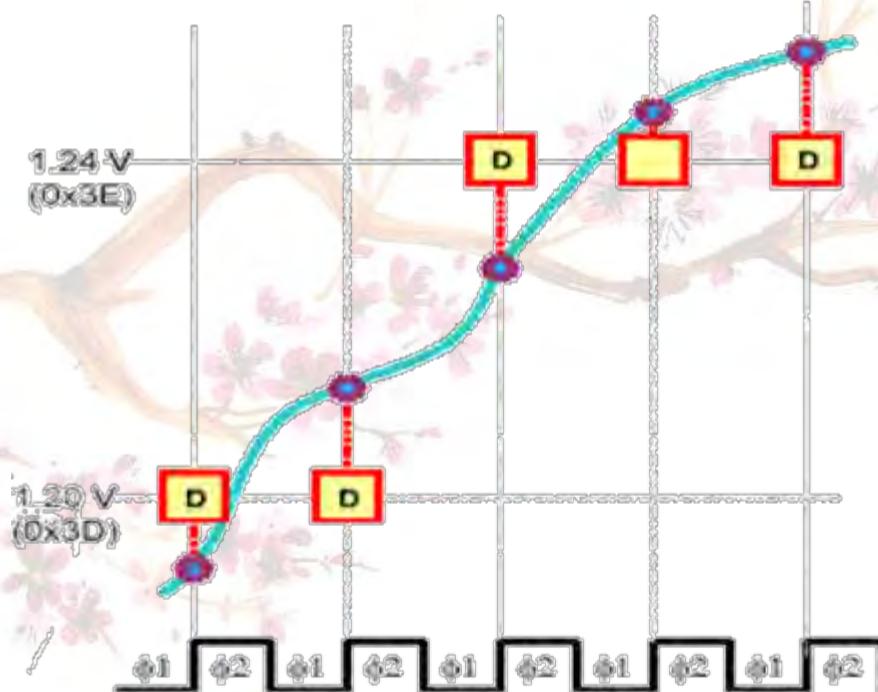


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Experimental implementation

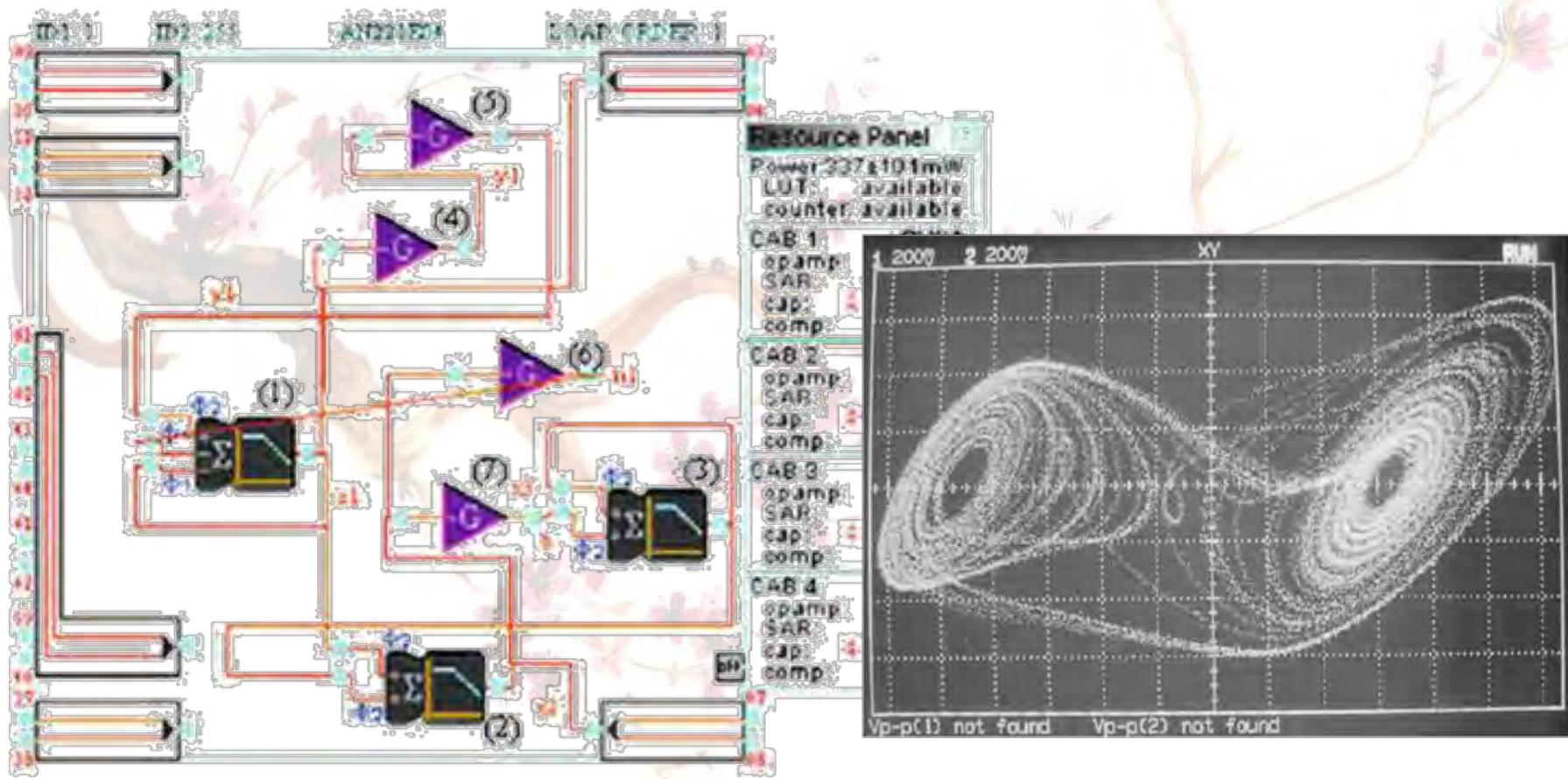
Sampled Analog is not digital!

Continuous-value,
discrete-time



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Experimental implementation



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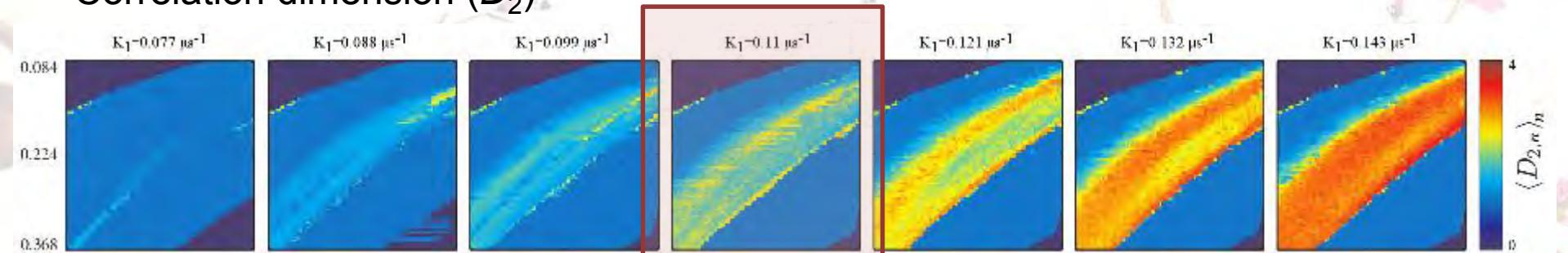
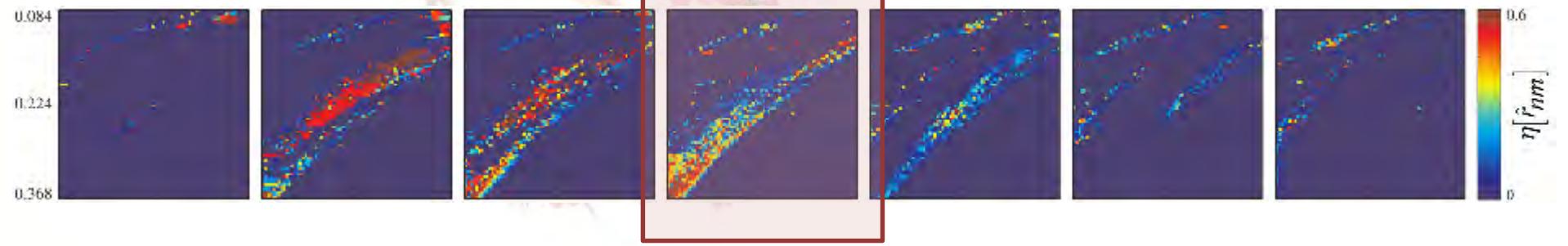
Experimental implementation



A soft of “Chimera”: an analog
plug-in system for digital computer

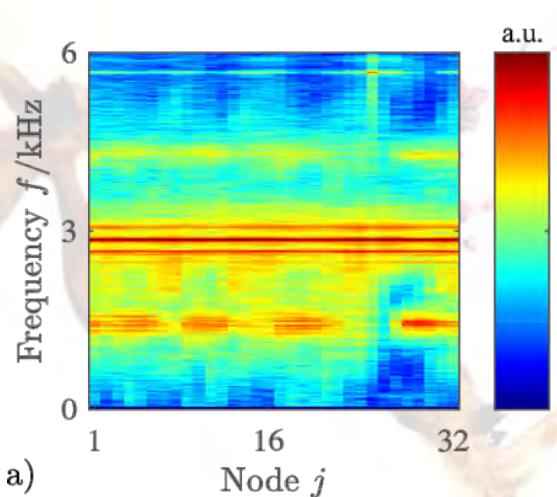
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Remote sync. close to quasi-periodicity

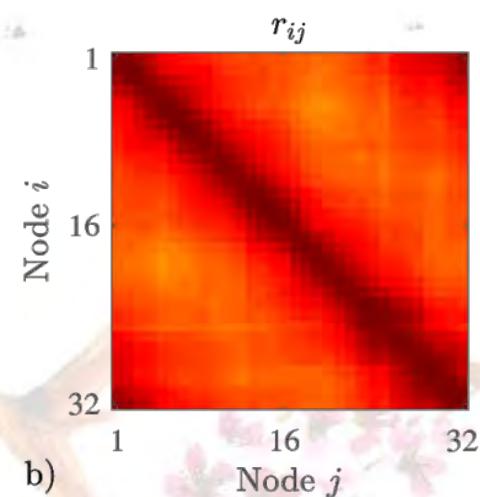
Correlation dimension (D_2)Remote synchronization (η)

Experimental data - basics

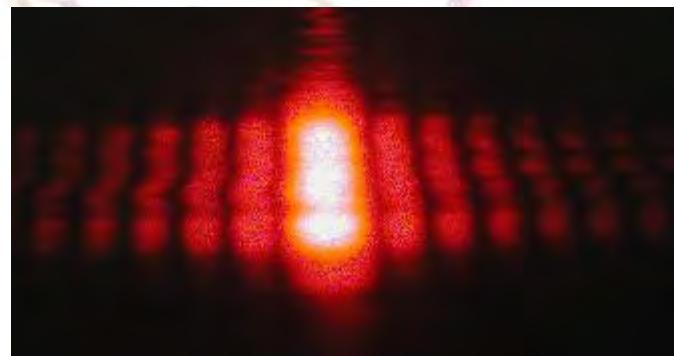
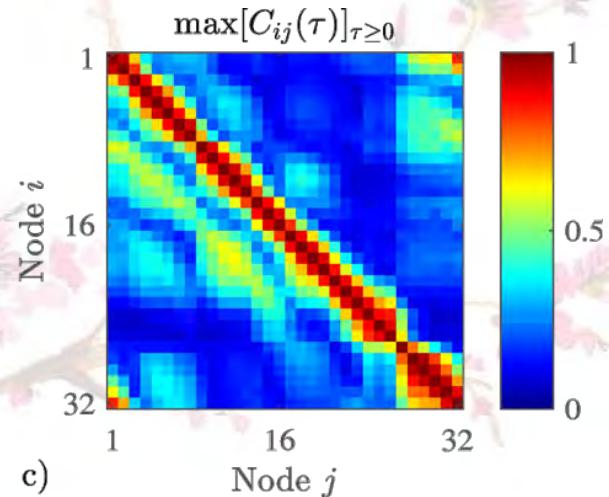
Spectrogram



Phase sync.



Amplitude sync.

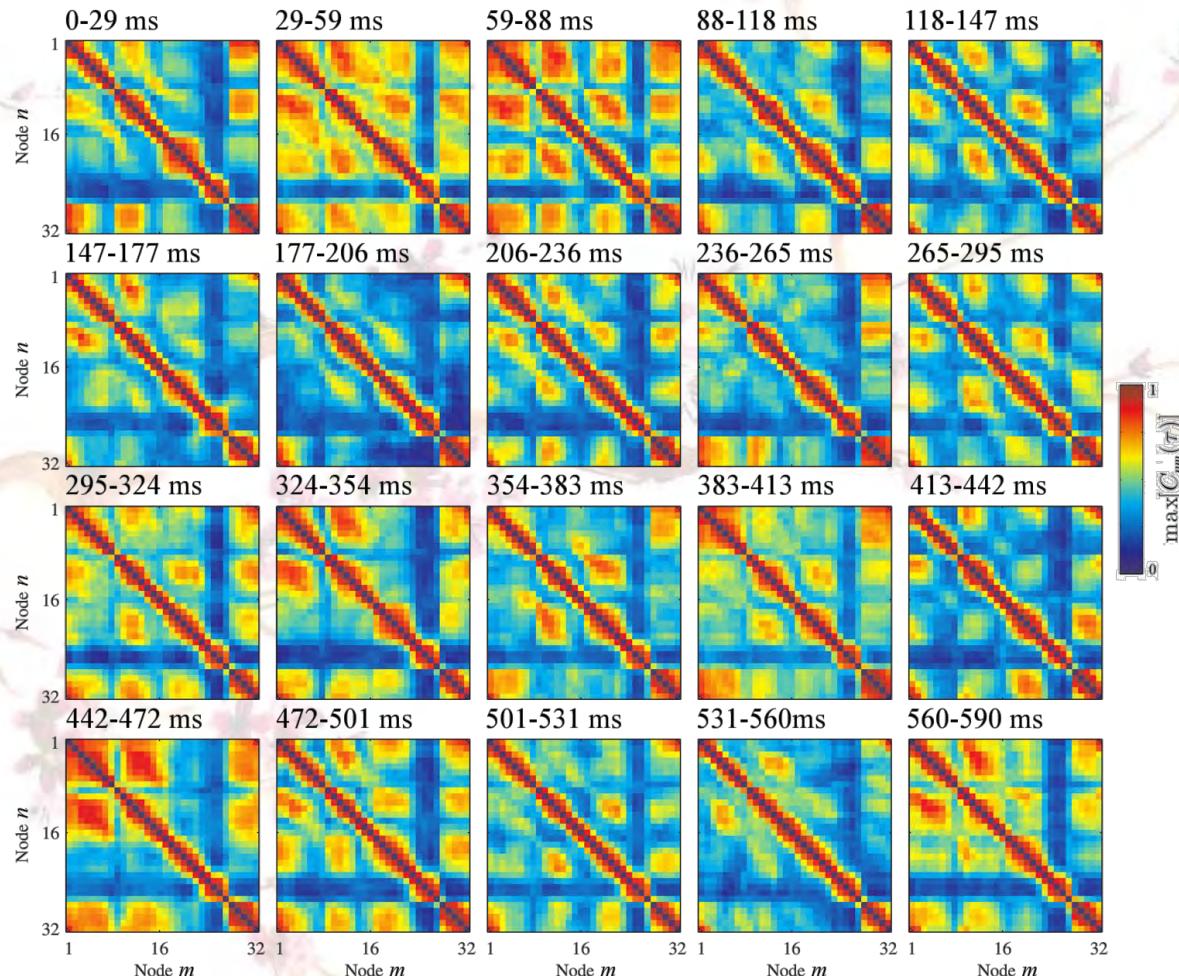


Reminiscent of a
diffraction pattern?

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Non-stationarity

Adjacent
time-windows...

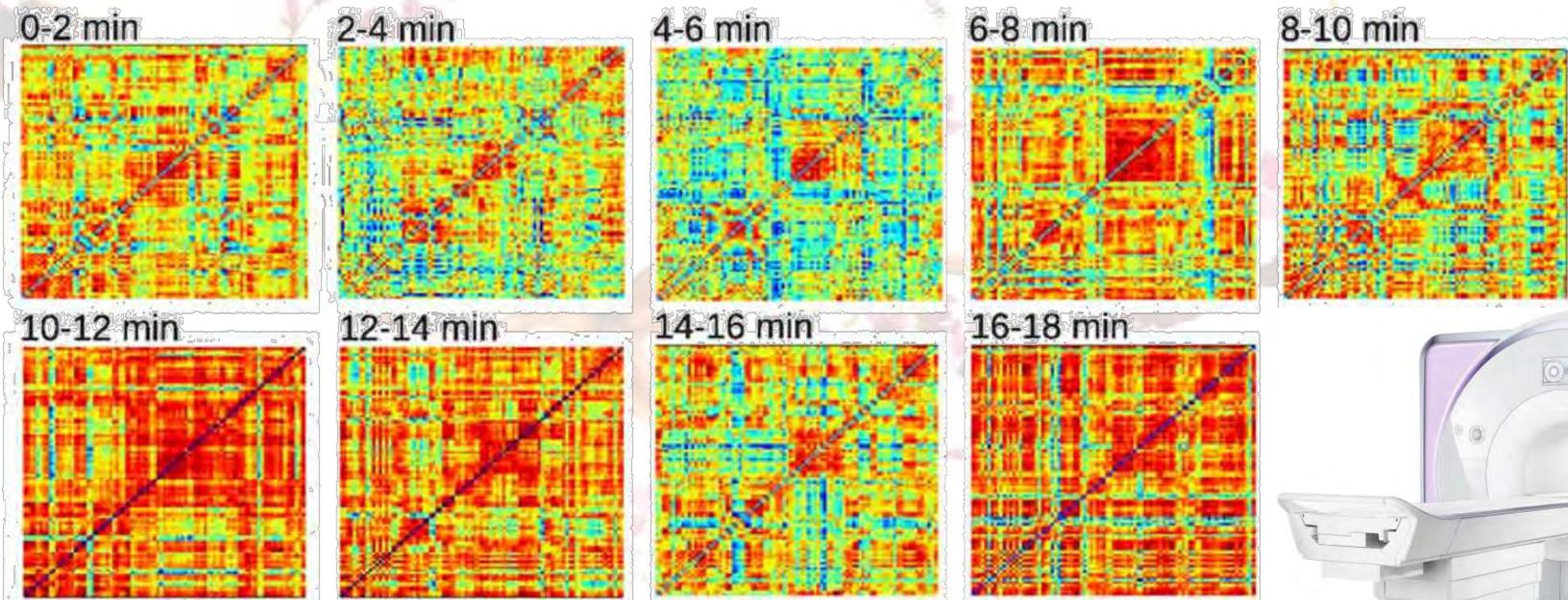


Remote synchronization: detailed account of a peculiar pattern-formation mechanism

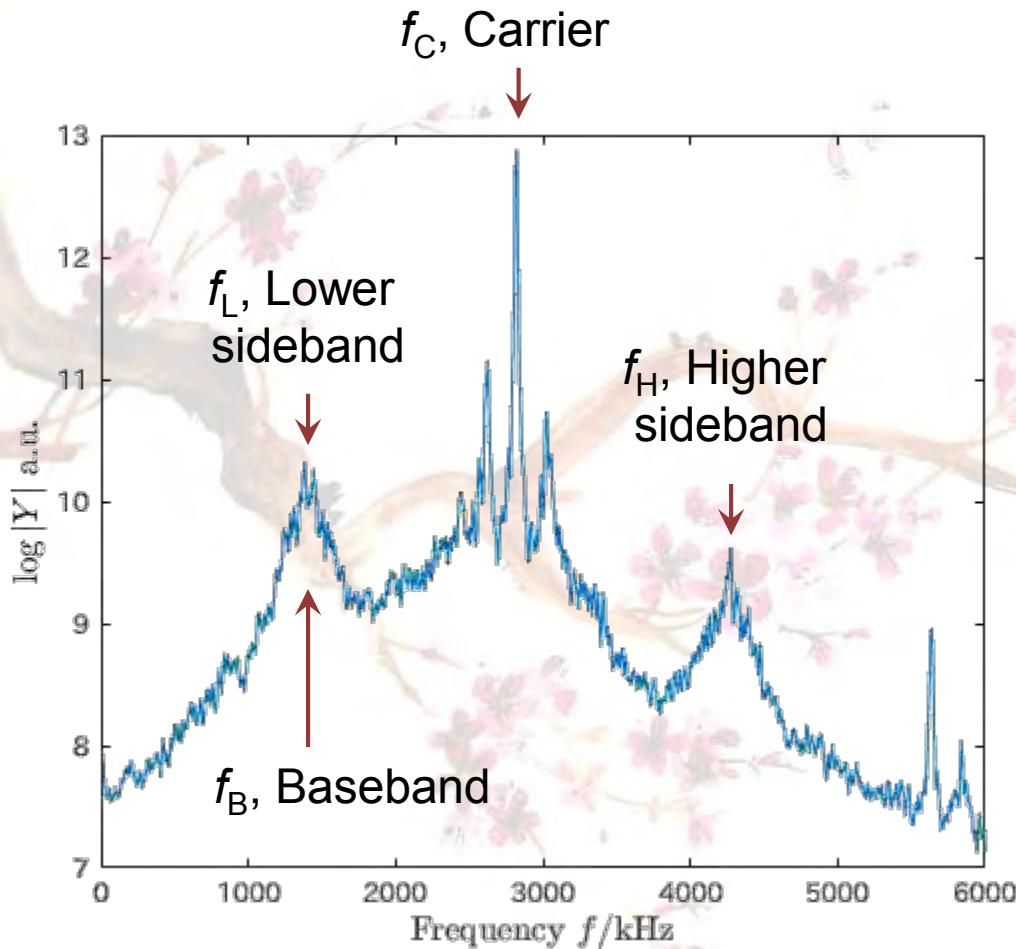
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Non-stationarity

...reminiscent of observations in resting-state functional MRI



Experimental data – focus on spectrum



Lower sideband and baseband overlap!

Two concomitant spectral relations:

$$1) f_B = f_H - f_C = f_C - f_L$$

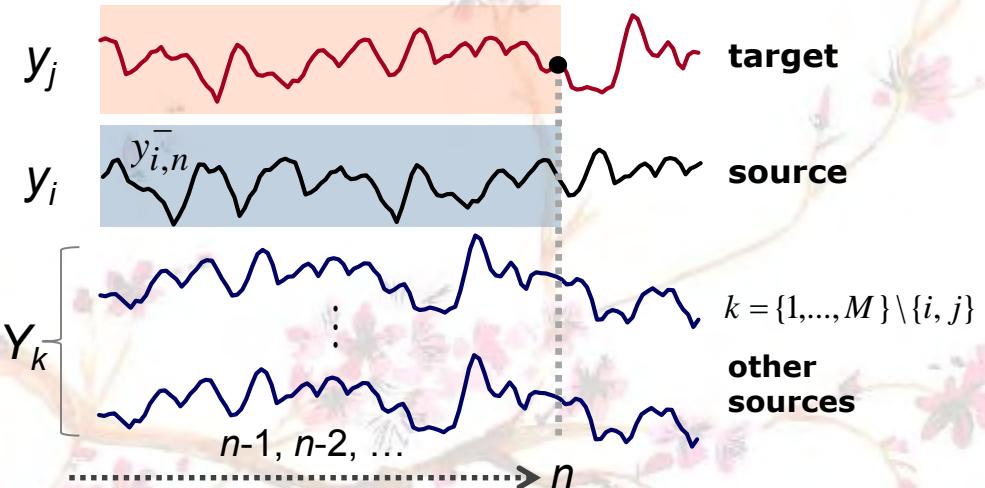
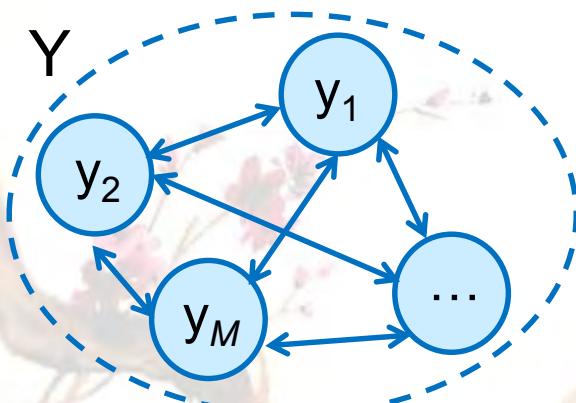
$$2) f_H = f_L + f_C \rightarrow f_L = f_B$$

Reminiscent of classic AM modulation!



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From synchronization to causality



- Regression of the present of the target on its own past:

$$e_{j|j,n} = y_{j,n} - E[y_{j,n} | y_{j,n}^-] \rightarrow \lambda_{j|j} = E[e_{j|j,n}^2]$$

- Regression of the present of the target on its past and the past of the source:

$$e_{j|ji,n} = y_{j,n} - E[y_{j,n} | y_{j,n}^-, y_{i,n}^-] \rightarrow \lambda_{j|ji} = E[e_{j|ji,n}^2]$$

Granger causality (GC)

$$F_{i \rightarrow j} = \ln \frac{\lambda_{j|j}}{\lambda_{j|ji}}$$

Gaussian

Transfer Entropy (TE)

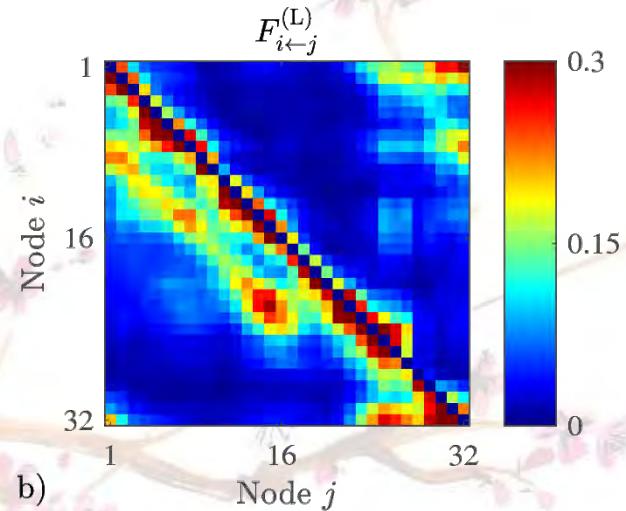
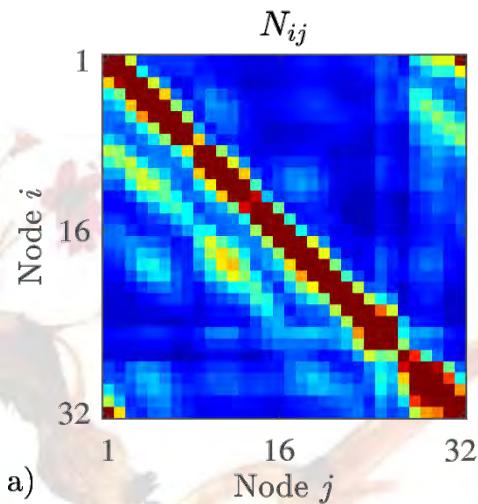
$$T_{i \rightarrow j} = \frac{1}{2} \ln \frac{\lambda_{j|j}}{\lambda_{j|ji}}$$

[J.F. Geweke, J. AM. Stat. Assoc. 77, 1982]

[L. Barnett et al., Phys. Rev. Lett. 103, 2009]

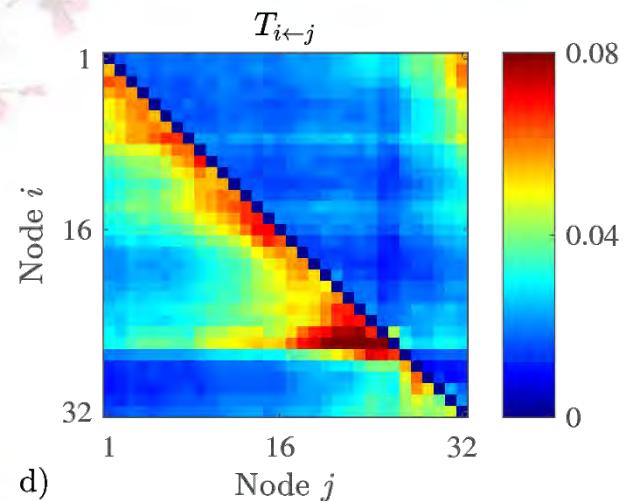
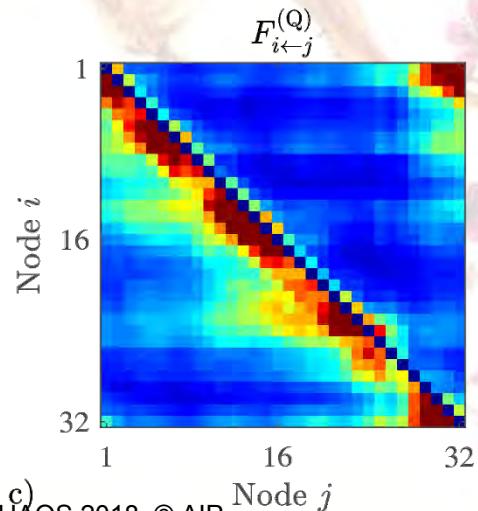
Mutual information and causality

Mutual
info.



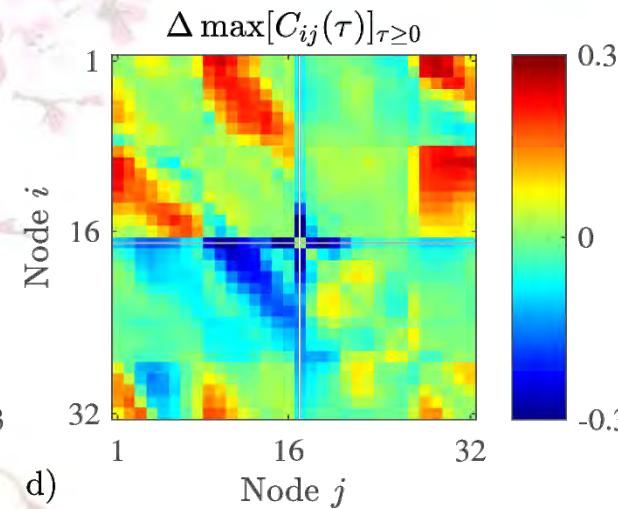
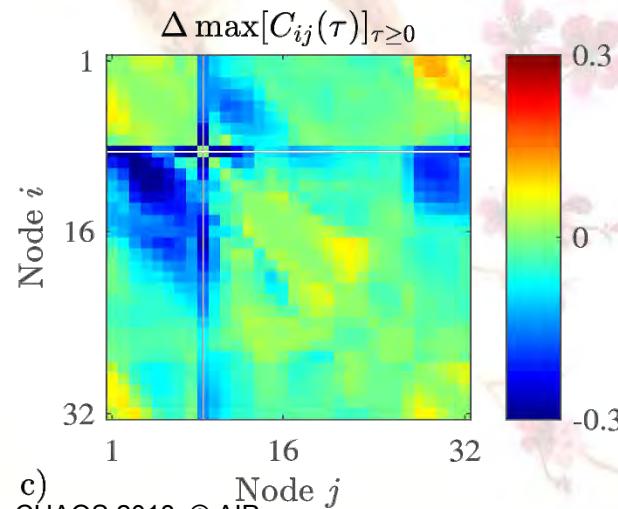
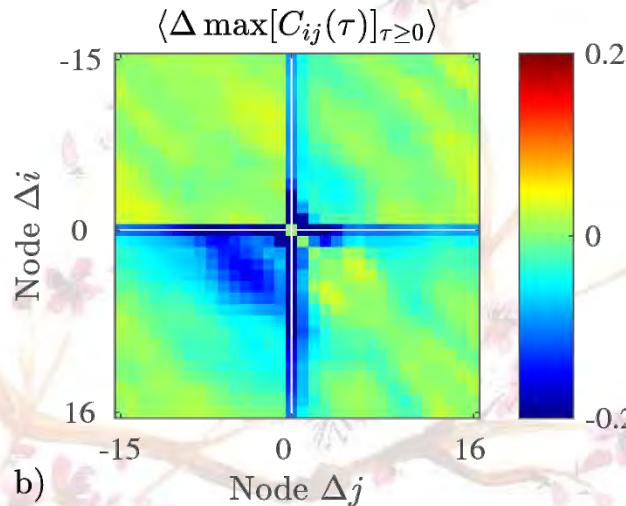
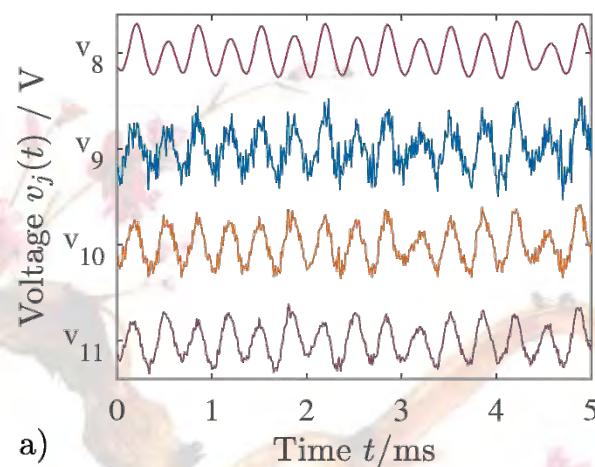
Linear
Granger

Granger
with quadratic+
cross-terms

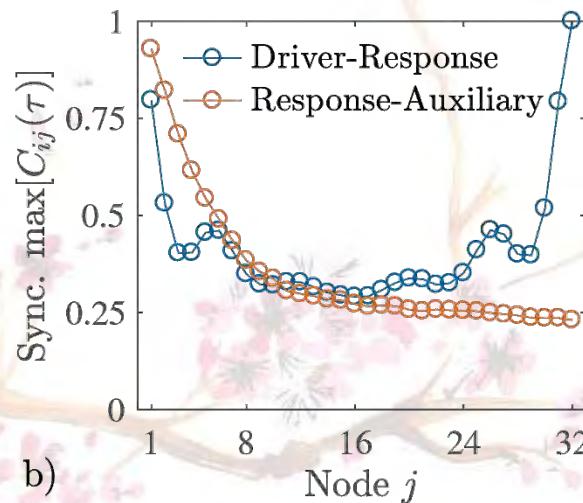
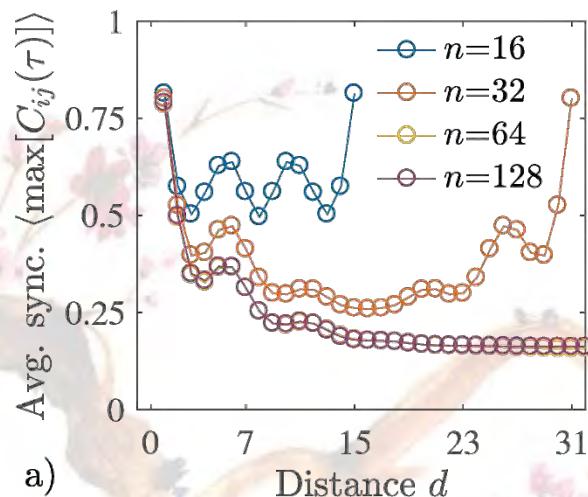


Transfer
Entropy

Effect of “lesioning” by noise injection

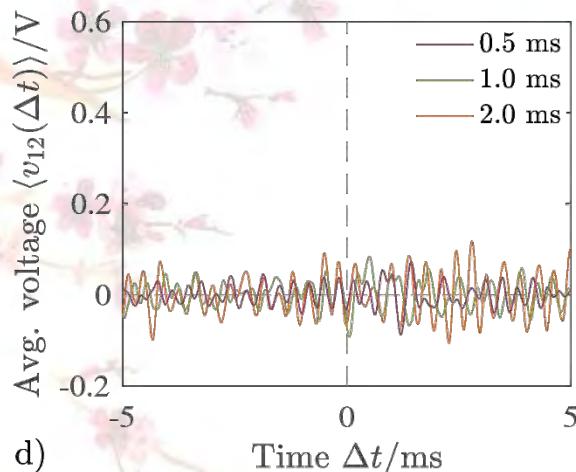
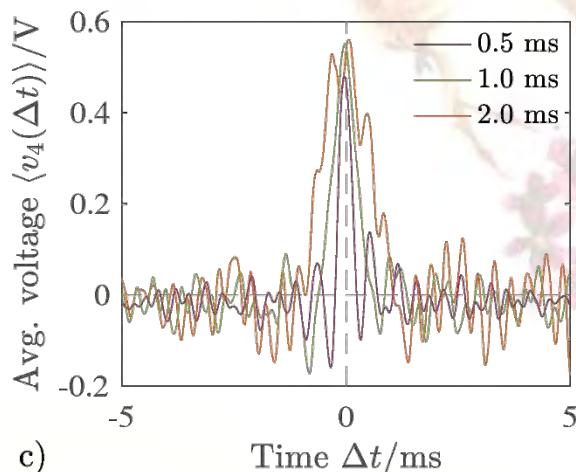
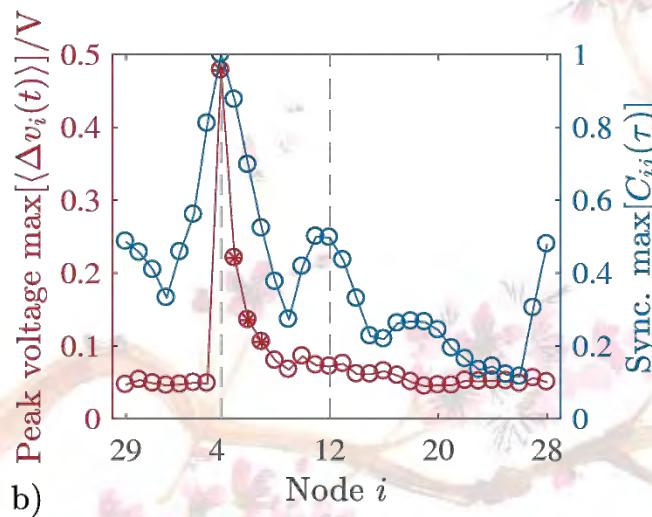
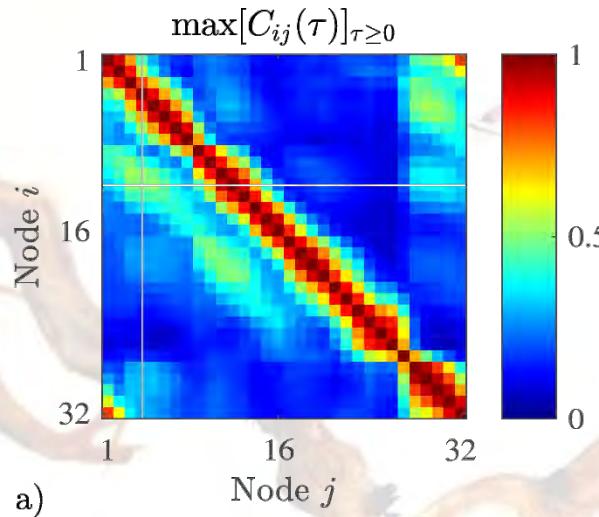


Ring size and auxiliary system simulations



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Propagation of external perturbations



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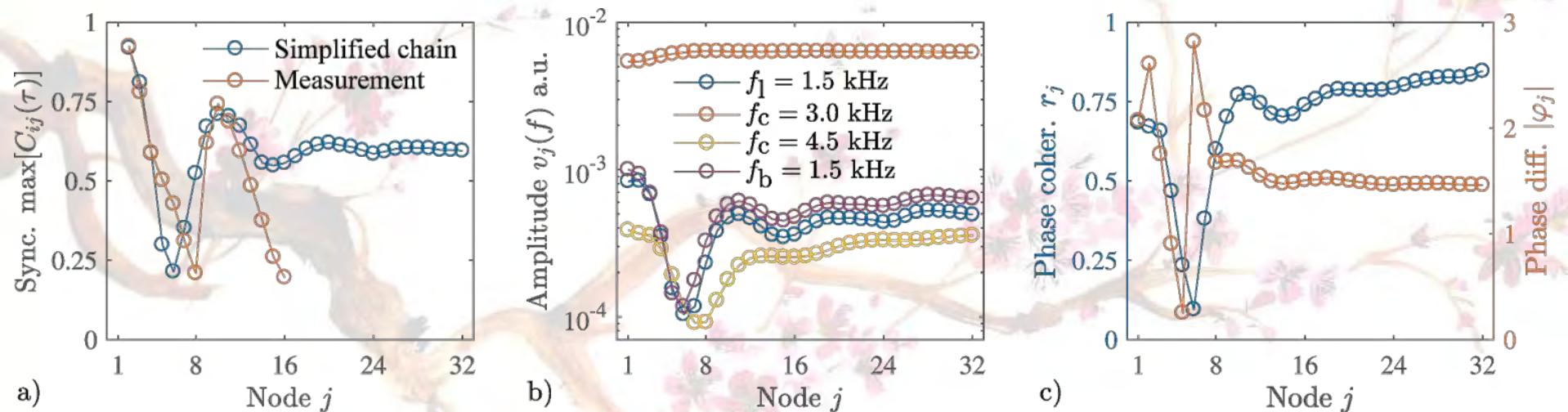
Simplified chain model

- 1) An open network is considered in the form of a chain.
- 2) Two dynamical equations are removed.
- 3) $\Gamma(x,y)$ is removed for all voltages except v_3 .
- 4) The parameters are set identically across all nodes.

$$\left\{ \begin{array}{l} \frac{dv_1}{dt} = 2\pi F(G_4v_4 - v_1) \\ \frac{dv_2}{dt} = 2\pi F(G_1v_0 - v_2) \\ \frac{dv_3}{dt} = \Gamma(Kv_0, v_3) \\ \frac{dv_4}{dt} = 2\pi F(G_2v_2 + G_3v_3 - v_4) \end{array} \right.$$

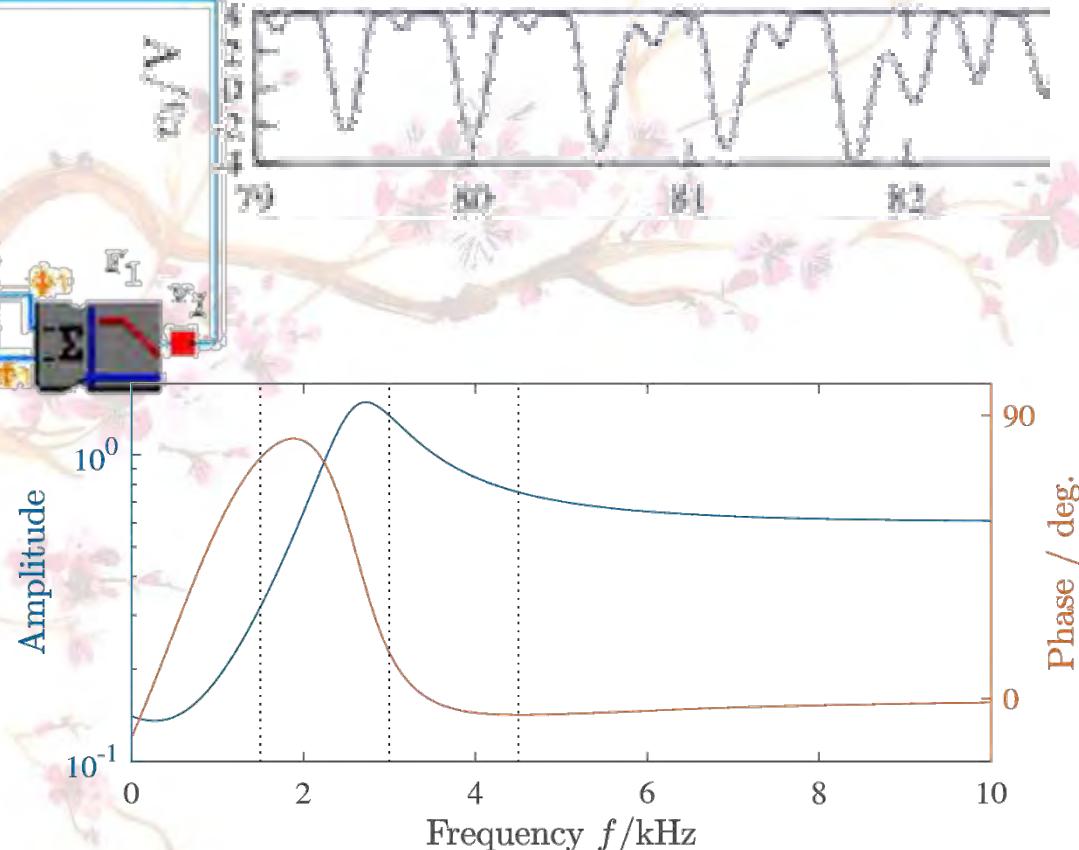
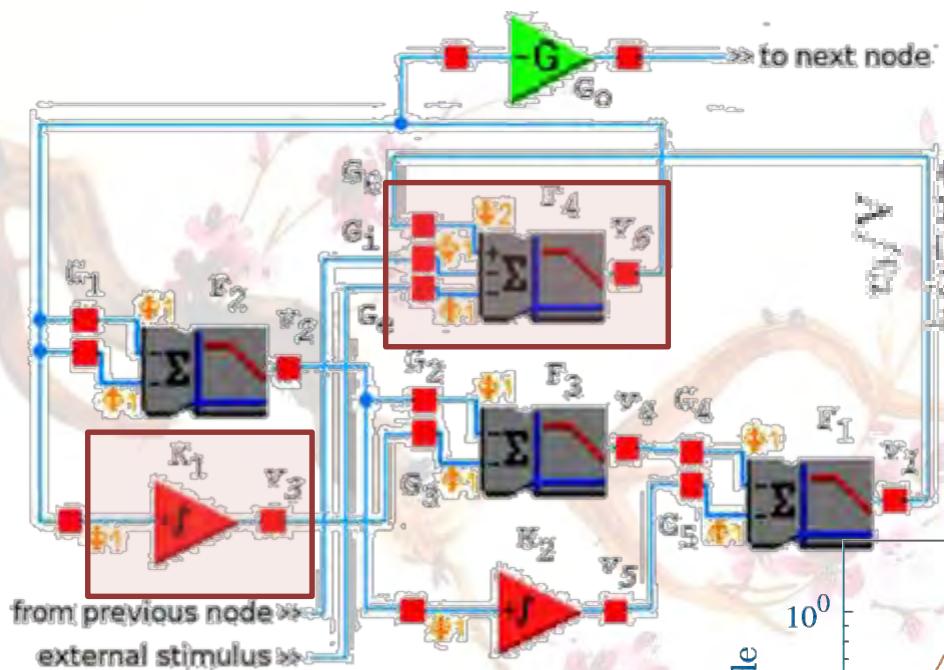
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Simplified chain model

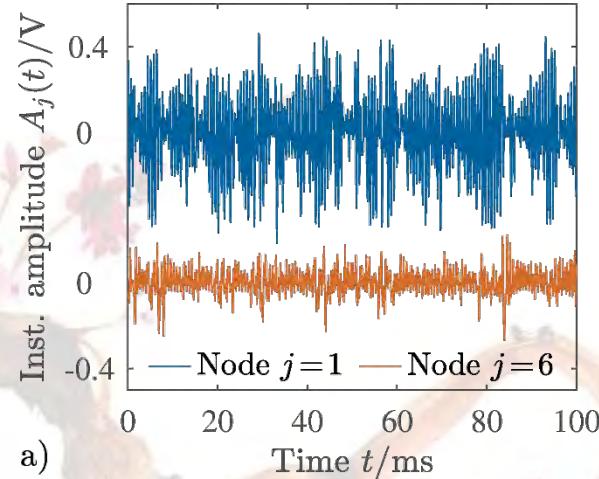


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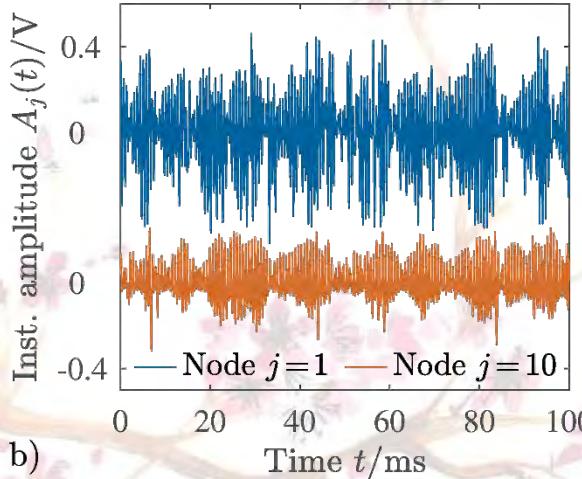
Demodulation and interference



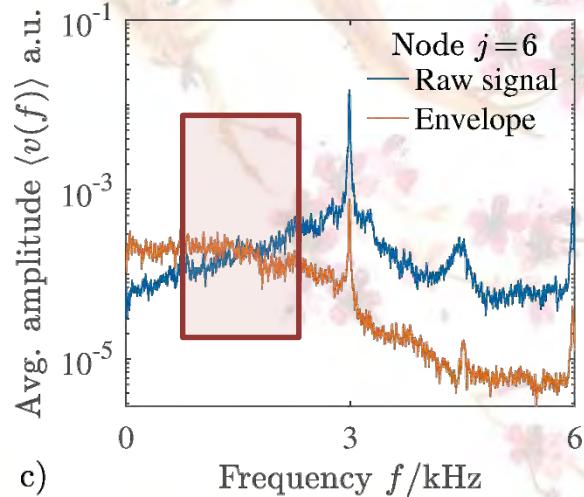
Demodulation and interference



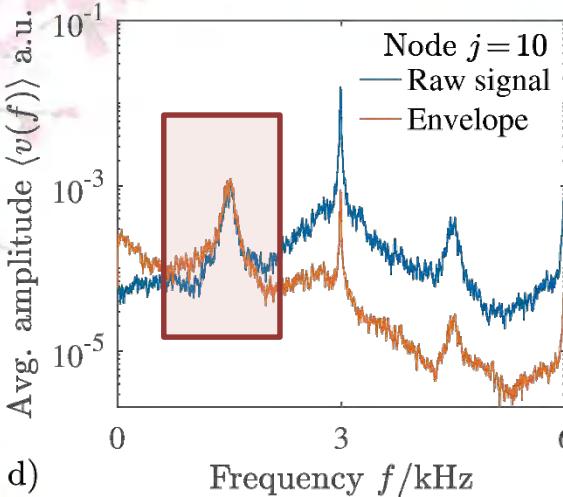
a)



b)



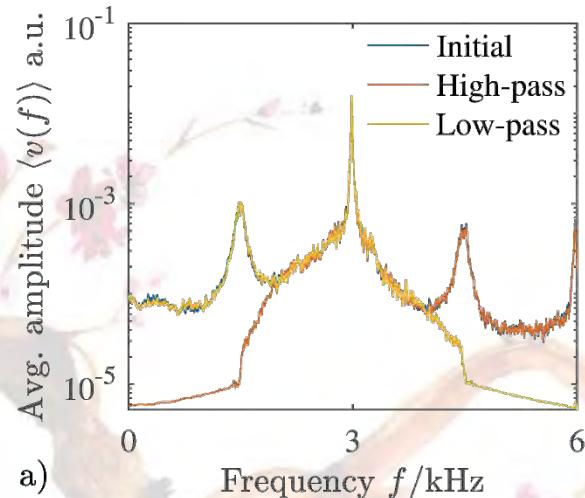
c)



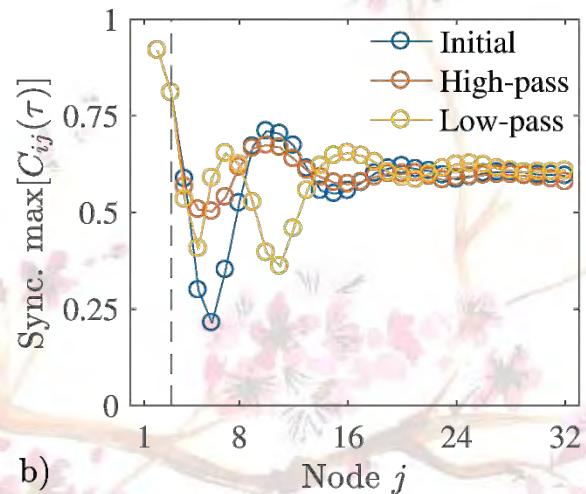
d)

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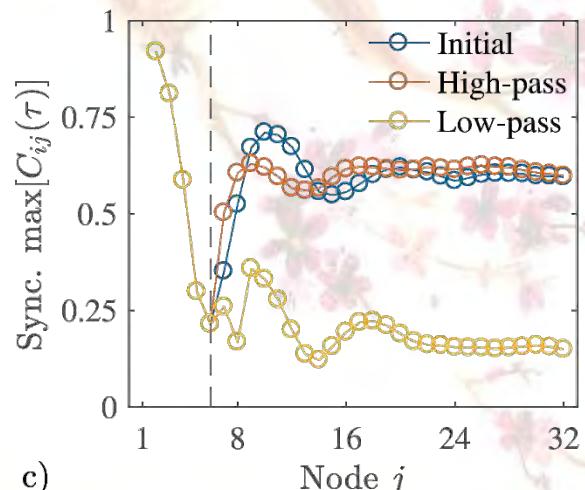
Instancing filters at specific points of chain



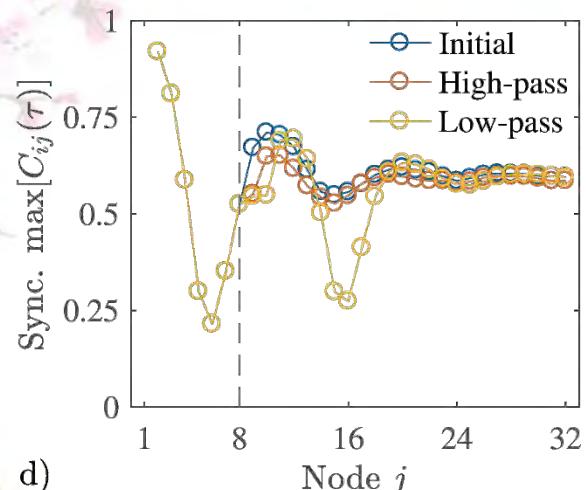
a)



b)



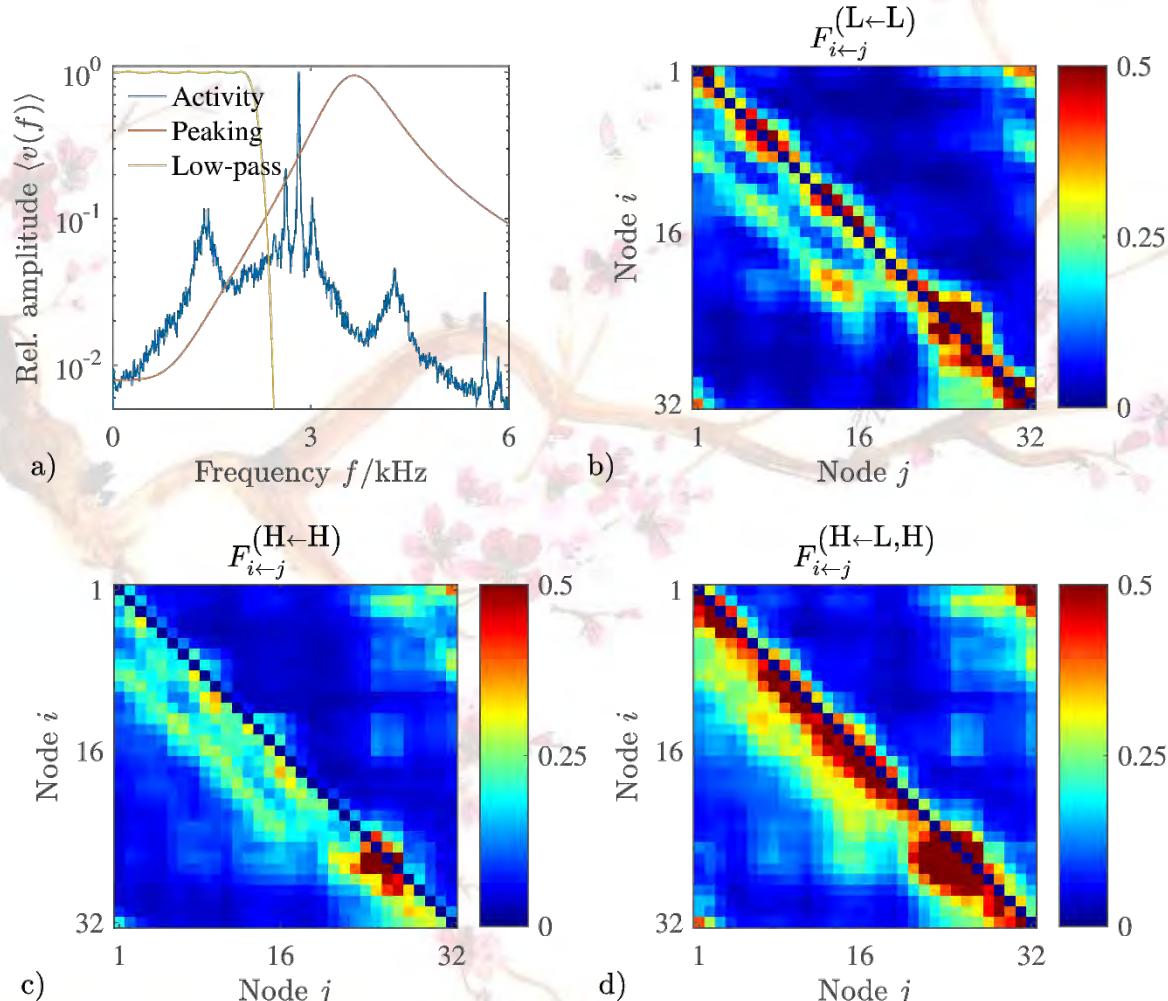
c)



d)

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Revised Granger model: baseband + sideband





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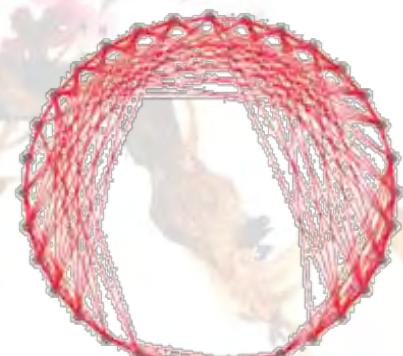
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*Remote synchronization: detailed account of a
peculiar pattern-formation mechanism*

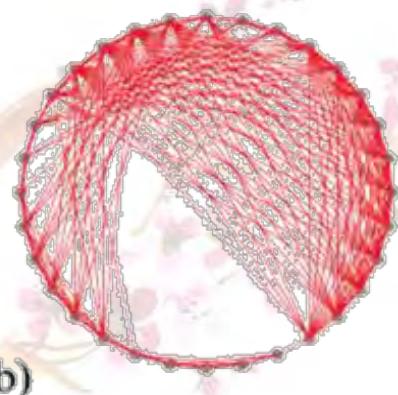
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Small-world features, nonetheless...

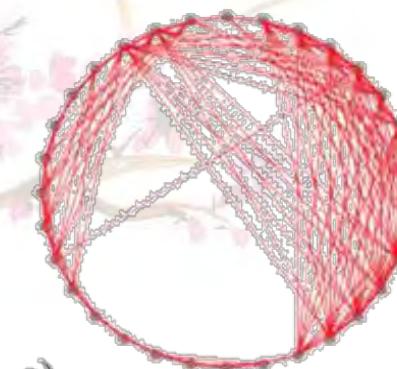
$$S^{\text{WS}} = \gamma_g^{\text{WS}} / \lambda_g = (C_g^{\text{WS}} L_{\text{rand}}) / (C_{\text{rand}}^{\text{WS}} L_g)$$



a)



b)



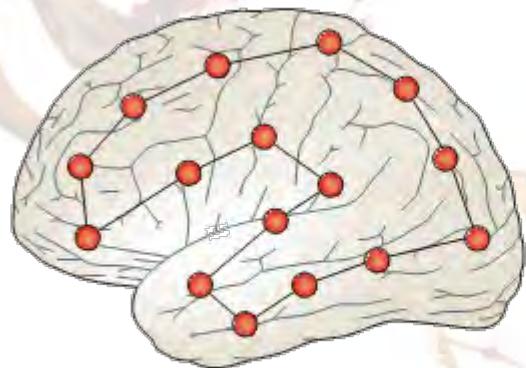
c)

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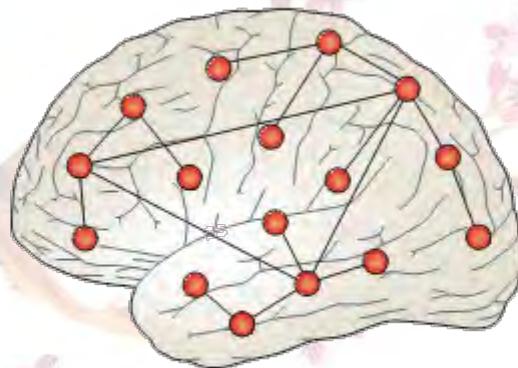
Small-world features, nonetheless...

Small-worldness in the brain (and not only) is an efficient trade-off!

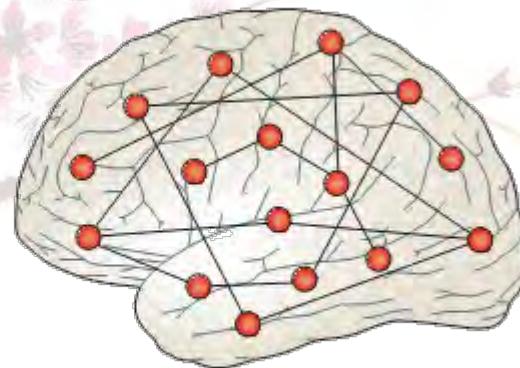
Lattice topology



Complex topology



Random topology



Cost

Low



High

Efficiency

Conclusions

- 1) A complex mechanism of pattern generation was demonstrated.
- 2) Is this just “apparent” remoteness?
Central importance of measure choice...
- 3) To what systems may such mechanism apply?
Broadband vs. narrowband chaos, spectral relationships

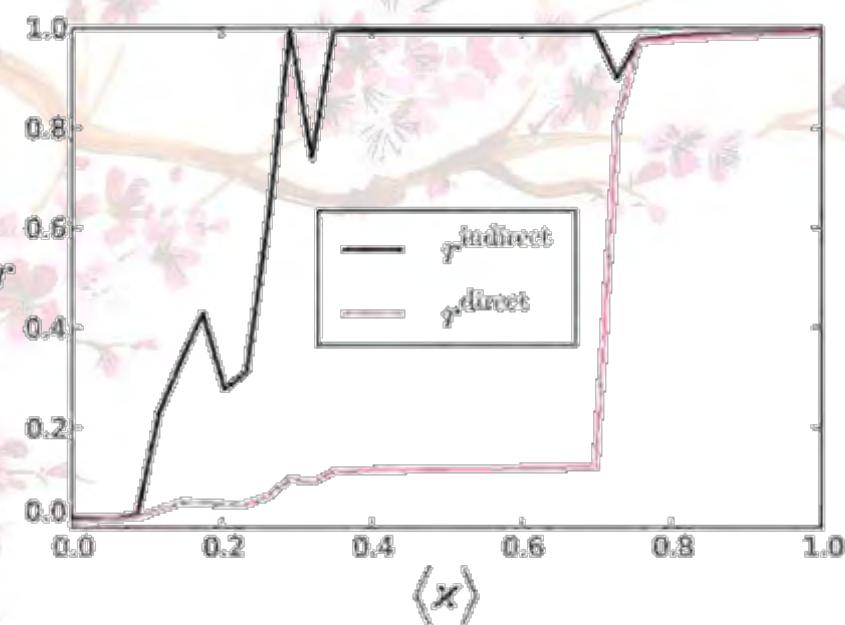
Remote synchronization from mismatches

PHYSICAL REVIEW E 85, 026208 (2012)

Remote synchronization in star networks

A. Bergner,^{1,3} M. Frasca,² G. Sciuto,² A. Buscarino,² E. J. Ngamga,³ L. Fortuna,² and J. Kurths^{3,4,5}

$$\dot{u}_i = (\alpha + i\omega_i - |u_i|^2)u_i + \frac{\sigma}{d_i} \sum_{j=1}^N a_{ij}(u_j - u_i)$$



Non-monotonic effect of the coupling strength

PHYSICAL REVIEW E 99, 052301 (2019)

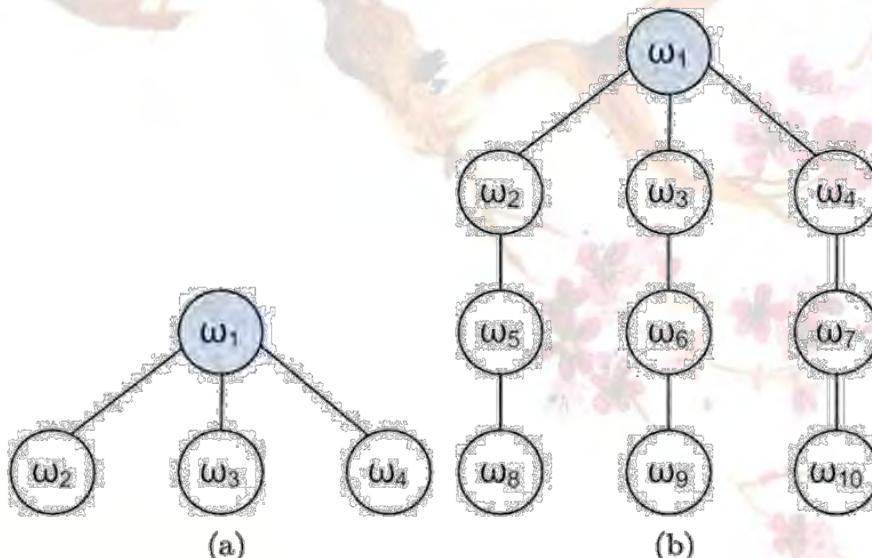
Fading of remote synchronization in tree networks of Stuart-Landau oscillators

Baris Karakaya,¹ Ludovico Minati,² Lucia Valentina Gambuzza,³ and Mattia Frasca^{3,4,*}

¹Faculty of Engineering, Department of Electrical, Electronics Engineering, Firat University, 23119 Elazig, Turkey

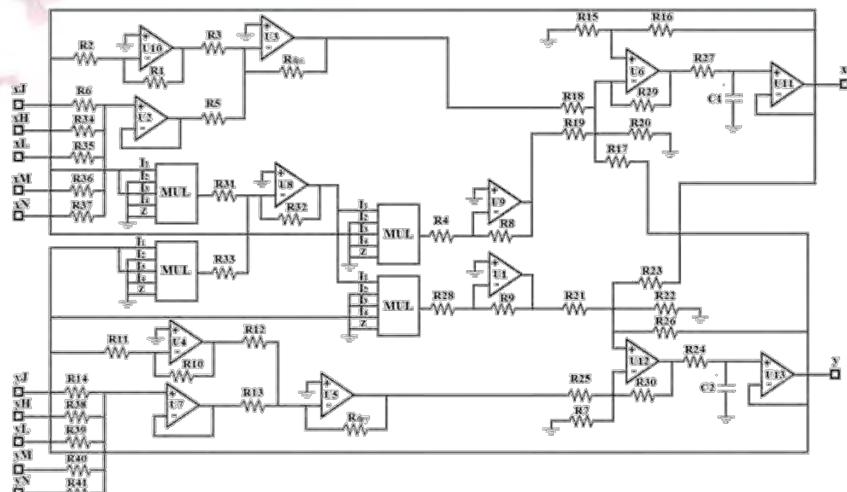
²World Research Hub Initiative—Institute of Innovative Research, Tokyo Institute of Technology, Yokohama 226-8503, Japan;

³Theoretical Nuclear Physics Department, Institute of Nuclear Physics, Polish Academy of Sciences, IFJ PAN, 21-222 Kraków, Poland.



$$\dot{x}_i = \alpha x_i - \omega_i y_i - x_i(x_i^2 + y_i^2) + \frac{\sigma}{k_i} \sum_{j=1}^N a_{ij}(x_j - x_i),$$

$$\dot{y}_i = \omega_i x_i + \alpha y_i - y_i(x_i^2 + y_i^2) + \frac{\sigma}{k_i} \sum_{j=1}^N a_{ij}(y_j - y_i),$$

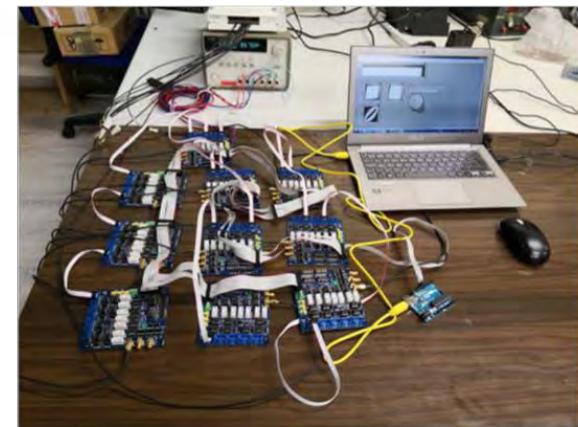
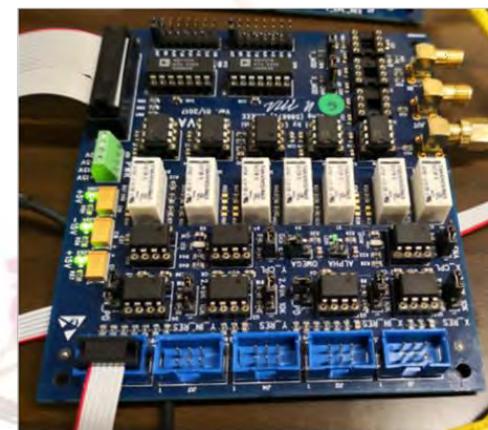
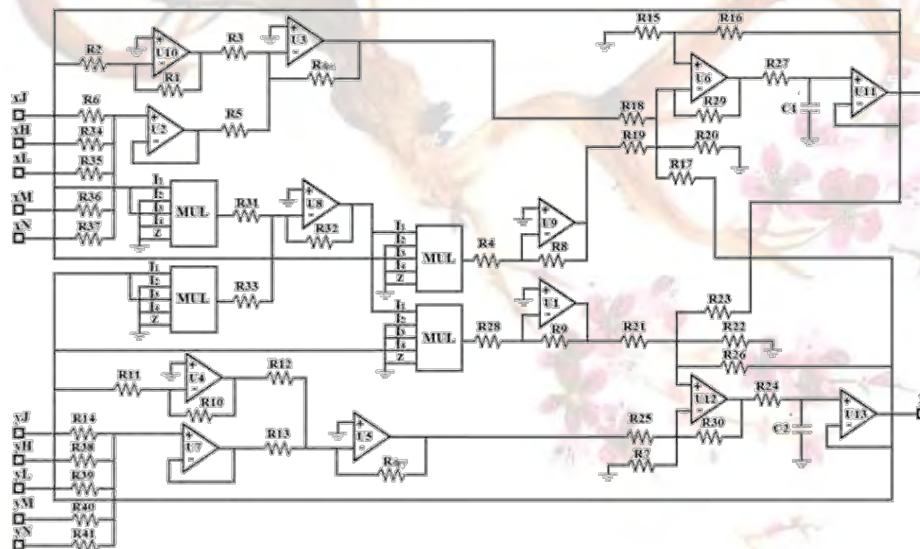


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Non-monotonic effect of the coupling strength

$$\dot{x}_i = \alpha x_i - \omega_i y_i - x_i(x_i^2 + y_i^2) + \frac{\sigma}{k_i} \sum_{j=1}^N a_{ij}(x_j - x_i),$$

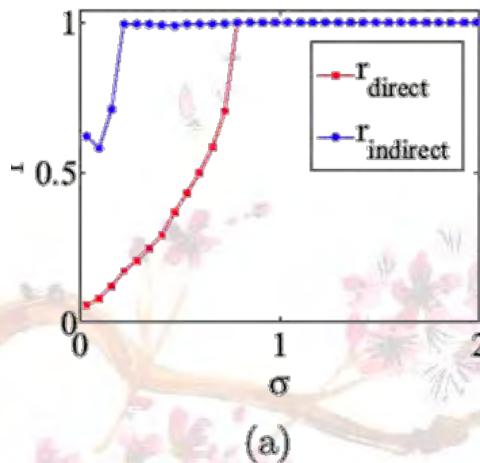
$$\dot{y}_i = \omega_i x_i + \alpha y_i - y_i(x_i^2 + y_i^2) + \frac{\sigma}{k_i} \sum_{j=1}^N a_{ij}(y_j - y_i),$$



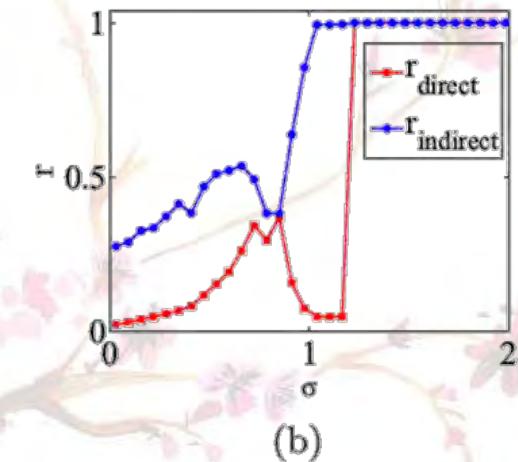
Non-monotonic effect of the coupling strength

$$r_{\text{indirect}} = \frac{2}{(N-1)(N-2)} \sum_{i=2, j>i}^N r_{ij}.$$

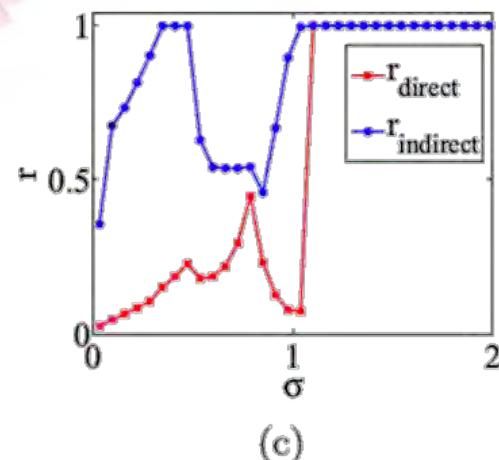
$$r_{\text{direct}} = \frac{1}{(N-1)} \sum_{j=2}^N r_{1j}.$$



(a)



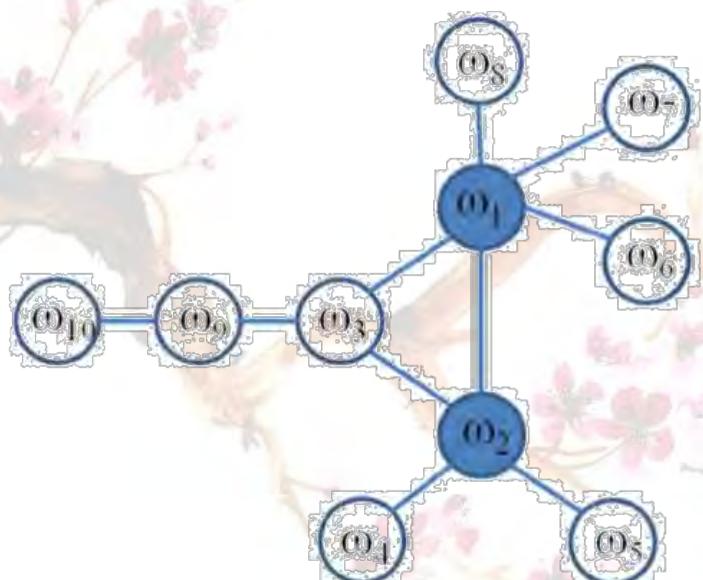
(b)



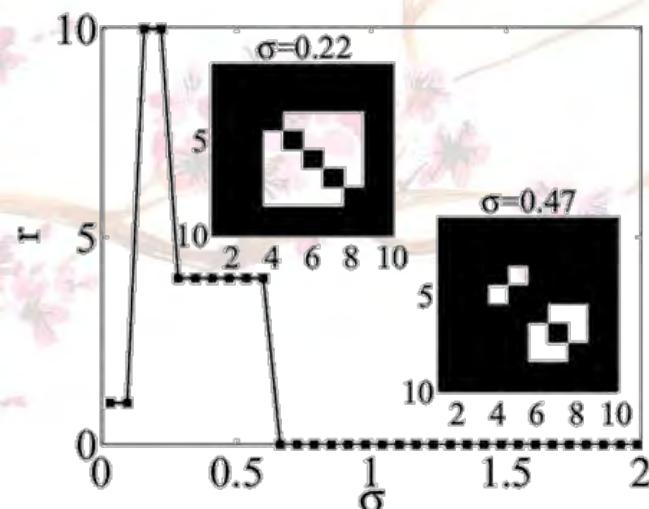
(c)

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Non-monotonic effect of the coupling strength



(a)



(b)



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Thank you for your attention

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1. Minati L. Remote synchronization of amplitudes across an experimental ring of non-linear oscillators. CHAOS. 2015 Dec; 25(12):123107.
2. Minati L, Faes L, Frasca M, Oświęcimka P, Drożdż S. Apparent remote synchronization of amplitudes: a demodulation and interference effect. CHAOS. 2018 Jun; 28, 063124.
3. Minati L, Across Neurons and Silicon: Some Experiments Regarding the Pervasiveness of Nonlinear Phenomena. Acta Phys Pol B 2018; 49:2029.
4. Karakaya B, Minati L, Gambuzza LV, Frasca M. Fading of remote synchronization in tree networks of Stuart-Landau oscillators. Phys Rev E. 2019; 99:052301.

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